

$$\lim_{n \rightarrow \infty} \frac{n^2 - 1}{n^2 + a + b} = -1 \Rightarrow \varepsilon + 2a + b = 0^+ \Rightarrow$$

$$n^2 + a + b = (n - 1)^2 = n^2 + \varepsilon - \varepsilon n \rightarrow a = -\varepsilon \text{ و } b = \varepsilon$$

$$a + b = \varepsilon - \varepsilon = 0 \quad \checkmark$$

$$\lim_{n \rightarrow \infty} \frac{n}{n^2 + a + b} = +\infty \Rightarrow \sqrt[3]{\varepsilon} \rightarrow n^2 + a + b = (n - \sqrt[3]{\varepsilon})^2$$

$$= n^2 + \sqrt[3]{1\varepsilon} - 2\sqrt[3]{\varepsilon}n = n^2 + a + b \rightarrow a = -2\sqrt[3]{\varepsilon} \text{ و } b = \sqrt[3]{\varepsilon}$$

$$\left[\frac{b}{a} \right] = \left[\frac{\sqrt[3]{1\varepsilon}}{-2\sqrt[3]{\varepsilon}} \right] = \left[\frac{-1}{2} \right] = -1 \quad \checkmark$$

$$\lim_{n \rightarrow \infty} [-n] + a = -\infty \quad n > \frac{1}{\sqrt{\varepsilon}} \rightarrow -n < -\frac{1}{\sqrt{\varepsilon}} \rightarrow [-n] = -1$$

$$\left| \frac{1}{\sqrt{\varepsilon}} + a \right| + a = 0^+ \rightarrow a < 0$$

$$a - 1 > 0 \rightarrow a > 1$$

$$\left| \frac{1}{\sqrt{\varepsilon}} + a \right| + a < 0 \rightarrow a < 0$$

$$a - 1 < 0 \rightarrow a < 1$$

$$\left| \frac{1}{\sqrt{\varepsilon}} + a \right| + a > 0 \rightarrow a > -\frac{1}{\sqrt{\varepsilon}}$$

$$[a] = -1 \quad \checkmark$$

$$n > \frac{1}{\sqrt{\varepsilon}} \rightarrow n^2 < \frac{1}{\varepsilon} \rightarrow \frac{1}{n^2} > \varepsilon \rightarrow \frac{-1}{n^2} < -\varepsilon \rightarrow \left[\frac{-1}{n^2} \right] = -1$$

$$n > \frac{1}{\sqrt{\varepsilon}} \rightarrow n^2 < \frac{1}{\varepsilon} \rightarrow \frac{1}{n^2} > \varepsilon \rightarrow \frac{1}{n^2} > 1 \rightarrow \left[\frac{1}{n^2} \right] = 1$$

$$\lim_{n \rightarrow \infty} \frac{12n + 9}{2\varepsilon n + 12} = \frac{-1 + 9}{-12 + 12} = \frac{1}{0^+} = +\infty$$

$$n > 1 \rightarrow n^2 > 1 \rightarrow [n^2] = 1 \quad \lim_{n \rightarrow \infty} \frac{n^2 - 1}{n^2 - 1} = \lim_{n \rightarrow \infty} \frac{(n - 1)(n + 1)}{(n - 1)(n^2 + \varepsilon + 2n)} =$$

$$\lim_{n \rightarrow \infty} \frac{n + 1}{n^2 + 2n + \varepsilon} = \frac{\varepsilon}{\varepsilon + \varepsilon + \varepsilon} = \frac{1}{3\varepsilon} \quad \checkmark$$

$$\lim_{x \rightarrow -1} \frac{x^2 + 10x + 15}{12 + \sqrt[5]{x}} \stackrel{HOP}{=} \lim_{x \rightarrow -1} \frac{x^2 + 10}{\frac{1}{\sqrt[5]{x}}} = \frac{(-1)^2 + 10}{\frac{1}{\sqrt[5]{-1}}} = -11 \quad \checkmark$$

(Y)

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1-\cos x}} = \frac{0}{0} \stackrel{HOP}{\rightarrow} \lim_{x \rightarrow 0^-} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$$

$$= \lim_{x \rightarrow 0^-} \frac{x}{\sqrt{1-\cos x}} = \lim_{x \rightarrow 0^-} \frac{x}{\frac{1}{2}x^2} \stackrel{HOP}{\rightarrow} \lim_{x \rightarrow 0^-} \frac{2}{x} = -\infty \quad \checkmark$$

(Y)

$$\lim_{x \rightarrow 0} \frac{K + \cos(\sqrt{a}x)}{Kx^2} = \frac{K+1}{0} = \infty \rightarrow K+1=0 \rightarrow K=-1$$

(I,VA)

$$\lim_{x \rightarrow 0} \frac{-1 + \cos(\sqrt{a}x)}{-x^2} \stackrel{HOP}{\rightarrow} \lim_{x \rightarrow 0} \frac{-1 + (1 - \frac{(\sqrt{a}x)^2}{2})}{-x^2} = \frac{-\frac{ax^2}{2}}{-x^2} = \frac{a}{2} = \infty$$

$$\frac{a}{K} = \frac{4}{-1} = -4$$

$$\rightarrow a = 4$$

$$\lim_{x \rightarrow (\sqrt{a})^+} \frac{\sqrt{x-a} + \sqrt{x} - \sqrt{x-a}}{\sqrt{x^2 - a^2}} = \lim_{x \rightarrow (\sqrt{a})^+} \frac{\sqrt{x-a}}{\sqrt{x^2 - a^2}} + \lim_{x \rightarrow (\sqrt{a})^+} \frac{\sqrt{x}}{\sqrt{x^2 - a^2}} = 0 + \frac{\sqrt{a}}{\sqrt{a^2}} = \frac{1}{\sqrt{a}} \quad \checkmark$$

$$\lim_{x \rightarrow (\sqrt{a})^+} \frac{\sqrt{x-a}}{\sqrt{x^2 - a^2}} \times \frac{\sqrt{x+a}}{\sqrt{x+a}} = \frac{\sqrt{x-a} \sqrt{x+a}}{\sqrt{x^2 - a^2} \sqrt{x+a}} = \frac{\sqrt{x^2 - a^2}}{\sqrt{x^2 - a^2} \sqrt{x+a}} = \frac{1}{\sqrt{x+a}}$$

$$\lim_{x \rightarrow (\sqrt{a})^+} \frac{\sqrt{x-a}}{\sqrt{x^2 - a^2}} \times \frac{\sqrt{x+a}}{\sqrt{x+a}} = \lim_{x \rightarrow (\sqrt{a})^+} \frac{\sqrt{x-a} \sqrt{x+a}}{\sqrt{x^2 - a^2} \sqrt{x+a}} = \lim_{x \rightarrow (\sqrt{a})^+} \frac{\sqrt{x^2 - a^2}}{\sqrt{x^2 - a^2} \sqrt{x+a}} = 0 \quad \checkmark$$

$$\lim_{x \rightarrow -1} \frac{1 - K[x]}{x^2 - 1} = -\infty \quad \begin{cases} \xrightarrow{-1^+} \lim_{x \rightarrow -1^+} \frac{1 - (Kx-1)}{x^2 - 1} = \lim_{x \rightarrow -1^+} \frac{1+K}{x^2 - 1} = -\infty \\ \xrightarrow{-1^-} \lim_{x \rightarrow -1^-} \frac{1 - (Kx-1)}{x^2 - 1} = \lim_{x \rightarrow -1^-} \frac{1+K}{x^2 - 1} = -\infty \end{cases}$$

$$\frac{x^2 - 1}{0^+} \rightarrow 1+K < 0 \rightarrow K < -1 \quad \checkmark$$

$$\lim_{x \rightarrow -1} \frac{1+K}{x^2 - 1} = -\infty \rightarrow 1+K > 0 \rightarrow K > -1$$

$$\Rightarrow -1 < K < -\frac{1}{2} \quad \begin{cases} \rightarrow \pi < K\pi < -\frac{\pi}{2} \\ \rightarrow -\frac{\pi}{2} < K\pi < 0 \end{cases} \Rightarrow \text{فصل ربعي} \quad \checkmark$$