ضمیمه تکمیلی دوازدهم دسته A

(1)

$$\lim_{x\to 2}\frac{2x-5}{x^2+ax+b}=-\infty \rightarrow x^2+ax+b=(x-2)^2=x^2-4x+4 \qquad \begin{cases}a=-4\\ b=4\end{cases}$$

$$a+b=-4+4=0$$

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(2)

$$\lim_{x\to\sqrt[3]{4}}\frac{x}{x^2+ax+b}=+\infty \rightarrow x^2+ax+b=(x-\sqrt[3]{4})^2 \rightarrow x^2-2\sqrt[3]{4}x+2$$

$$a=-2\sqrt[3]{4}$$

$$b=2$$

$$\left[\frac{b}{a}\right]=\left[\frac{2}{-2\sqrt[3]{4}}\right]=-1$$

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(3)

$$\lim_{x\to\left(\frac{1}{\sqrt[3]{2}}\right)^+}\frac{[-x]+a}{\left|\frac{\sqrt[3]{4}}{2}x+a\right|+a}=-\infty \qquad \left|\frac{\sqrt[3]{4}}{2}\times\frac{1}{\sqrt[3]{2}}+a\right|+a=0$$

$$2a=-\frac{1}{2} \rightarrow a=-\frac{1}{4} \qquad \left[-\frac{1}{4}\right]=-1$$

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(4)

$$\lim_{x\to\left(-\frac{1}{2}\right)^+}\frac{16x-\left[-\frac{2}{x^2}\right]}{24x+\left[\frac{3}{x^2}\right]}=\lim_{x\to\left(-\frac{1}{2}\right)^+}\frac{16x-4}{24x-12}=\frac{16}{24}=\frac{2}{3}$$

$$\left[-\frac{2}{x^2}\right]=[+4^+]=4$$

$$\left[\frac{3}{x^2}\right]=[-12^+]=-12$$

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(5)

$$\lim_{x\to 2^+}\frac{x^2-4}{x^3-[x^3]}=\lim\frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+2x+4)}=\frac{4}{12}=\frac{1}{3}$$

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(6)

$$\lim_{x\to -8}\frac{x^2+10x+16}{12+6\sqrt[3]{x}}=\frac{0}{0}\xrightarrow{Hop}\frac{2x+10}{6\left(\frac{1}{3\sqrt[3]{x^2}}\right)}=\frac{-6}{6\left(\frac{1}{12}\right)}=-12$$

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(7)

$$\lim_{x\to 0^-}\frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}}=\frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{\frac{x^2}{2}}}=\lim_{x\to 0^-}\frac{\sqrt{2+3x}-\sqrt{2-x}}{\frac{1}{\sqrt{2}}\underbrace{|x|}_{-x}}=\frac{0}{0}\xrightarrow{Hop}$$

$$\frac{\frac{3}{2\sqrt{2+3x}}+\frac{1}{2\sqrt{2-x}}}{\frac{-1}{\sqrt{2}}}=\frac{\frac{3}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}}{\frac{-1}{\sqrt{2}}}=\frac{\frac{4}{2\sqrt{2}}}{\frac{-1}{\sqrt{2}}}=-2$$

(11)

$$\lim_{x\to 0}\frac{k+\cos(\sqrt{a}\,x)}{kx^2}=3 \qquad \frac{a}{k}=?$$

$$\xrightarrow{Hop}\frac{-\sqrt{a}\,\sin(\sqrt{a}\,x)}{2kx}\xrightarrow{Hop}\frac{-a\cos(\sqrt{a}\,x)}{2k}=3$$

$$\lim_{x\to 0}\frac{-a\cos(\cancel{\sqrt{a}\,x})}{2k}=\frac{-a}{2k}=3\longrightarrow\frac{a}{k}=-6$$

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(6)

$$\lim_{x\to 4a^+}\frac{2\sqrt{x-4a}+3(\sqrt{x}-\sqrt{4a})}{\sqrt{x-4a}\times\sqrt{x+4a}}\longrightarrow\times\frac{\sqrt{x}+\sqrt{4a}}{\sqrt{x}+\sqrt{4a}}$$

$$\longrightarrow\lim_{x\to 4a^+}\frac{2\sqrt{x-4a}+3(x-4a)}{\sqrt{x-4a}\cdot\sqrt{x+4a}\,(\sqrt{x}+\sqrt{4a})}$$

$$\longrightarrow\lim\frac{\cancel{\sqrt{x-4a}}\left(2+3\cancel{(\sqrt{x-4a})}^{\,0}\right)}{\cancel{\sqrt{x-4a}}\cdot\underbrace{\sqrt{x+4a}\cdot(\sqrt{x}+\sqrt{4a})}_{\sqrt{8a}}}=\frac{2}{9\sqrt{2}a^2}$$

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(15)

$$\lim\frac{1-k[x]}{x^2-1}=-\infty$$


$$\lim_{x\to(-1)^+}\frac{1+k}{0^-}=-\infty\to 1+k>0\to k>-1$$

$$\lim\frac{1+2k}{0^+}=-\infty\to 1+2k<0\to k<-\frac{1}{2}$$

$$-1<k<-\frac{1}{2}$$

$$-\pi<k\pi<-\frac{\pi}{2}$$

$$-1<\cos k\pi<\cos-\frac{\pi}{2}$$