۴) مقدار $\lim\limits_{x \to \left(\frac{-1}{2}\right)^+} \dfrac{16x - \left[-\frac{2}{x^2}\right]}{24x + \left[\frac{3}{x^2}\right]}$ را بیابید.

$x > \frac{-1}{2} \xrightarrow{\text{توان } 2} 0 < x^2 < \frac{1}{4} \xrightarrow{\text{معکوس}} \frac{1}{x^2} > 4 \xrightarrow{\times(-2)} \frac{-2}{x^2} < -8 \rightarrow \left[\frac{-2}{x^2}\right] = -9$

$x > \frac{-1}{2} \rightarrow 0 < x^2 < \frac{1}{4} \longrightarrow 4 < \frac{1}{x^2} \xrightarrow{\times 3} 12 < \frac{3}{x^2} \rightarrow \left[\frac{3}{x^2}\right] = 12$

$\dfrac{16x + 9}{24x + 12} = \dfrac{1}{-12^+ + 12} = \dfrac{1}{0^+} = +\infty$

$\left(x > \frac{-1}{2} \xrightarrow{\times 24} 24x > -12\right)$

۵) حاصل $\lim\limits_{x \to 2^+} \dfrac{x^2 - 4}{x^3 - [x^3]}$ را بیابید.

$\dfrac{x^2 - 4}{x^3 - 8} = \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2 + 4 + 2x)} = \dfrac{4}{4+4+4} = \dfrac{1}{3}$

۶) حد عبارت $\dfrac{x^2 + 10x + 16}{12 + 6\sqrt[3]{x}}$ وقتی $x \to -8$ را بیابید.

$\frac{0}{0} \xrightarrow{\text{Hop}} \dfrac{2x + 10}{\frac{6}{3\sqrt[3]{x^2}}} = \dfrac{-6}{\frac{6}{12}} = \dfrac{-6}{\frac{1}{2}} = -12$

۷) حاصل $\lim\limits_{x \to 0^-} \dfrac{\sqrt{2+3x} - \sqrt{2-x}}{\sqrt{1 - \cos}}$ را بدست آورید.

$\frac{0}{0} \rightarrow \dfrac{\sqrt{2+3x} - \sqrt{2-x}}{\left|\sin\frac{x}{2}\right|} = \dfrac{\sqrt{2+3x} - \sqrt{2-x}}{-\sin\frac{x}{2}} \underset{\text{هم‌ارزی}}{=} \dfrac{\sqrt{2+3x} - \sqrt{2-x}}{-\frac{x}{2}} \underset{\text{هم‌ارزی}}{=} \dfrac{\frac{3}{2\sqrt{2+3x}} + \frac{1}{2\sqrt{2-x}}}{-1}$

$(-1)\left(\dfrac{3}{2\sqrt{2}} + \dfrac{1}{2\sqrt{2}}\right) = (-1)\left(\dfrac{\cancel{4}^{\,2}}{\cancel{2}\sqrt{2}}\right) = -\sqrt{2}$