نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۵ ... کلاس .....

۱

$$\lim_{x\to 2}\frac{x^2-5}{x^2+ax+b}=-\infty$$

$x^2-5 \xrightarrow{x=2} 2^2-5=-1 \longrightarrow$ مخرج $\to 0^+$ برای $2^+, 2^-$

$(x-2)^2=x^2+ax+b$

$a=-4$, $b=4$ $\Rightarrow a+b=0$

۲

$$\lim_{x\to \sqrt[3]{4}}\frac{x}{x^2+ax+b}=+\infty$$

$(x-\sqrt[3]{4})^2=x^2+ax+b$

$a=-\sqrt[3]{32}$, $b=\sqrt[3]{16}$ $\Rightarrow \left[\frac{b}{a}\right]=\left[\frac{-16}{\sqrt[3]{32}}\right]=-1$

مخرج $\to 0^+$ برای $\sqrt[3]{4}^-$ و $\sqrt[3]{4}^+$

$-1,0$ بین

۳

$$\lim_{x\to \left(\frac{1}{\sqrt[3]{2}}\right)^+}\frac{[-x]+a}{\left|\frac{\sqrt{3}}{3}x+a\right|+a}=-\infty$$

$x=\frac{1}{\sqrt[3]{2}} \to -x=\frac{-\sqrt[3]{2}}{3} \to [-x]=-1$

$\sqrt[3]{2}x=\frac{1}{3}$

$a>-\frac{1}{3} \to \frac{a-1}{2a+\frac{1}{3}}=-\infty \to a=\left(-\frac{1}{6}\right)^+ \to \frac{-\frac{7}{6}}{0^+}=-\infty$

$\frac{a-1}{a+\left|\frac{1}{3}+a\right|}$

$a<-\frac{1}{3} \to \frac{a-1}{a-\frac{1}{3}-a}=-3a$

$[a]=\left[-\frac{1}{6}\right]=-1$

۴

$$\lim_{x\to \left(-\frac{1}{2}\right)^+}\frac{|4x-\left[\frac{-2}{x^2}\right]|}{2[x]+\left[\frac{2}{x^2}\right]}$$

$x>-\frac{1}{2} \to x^2<\frac{1}{4} \to \frac{1}{x^2}>4$

$\frac{-2}{x^2}<-1 \to \left[\frac{-2}{x^2}\right]=-4$

$\frac{3}{x^2}>1 \to \left[\frac{3}{x^2}\right]=1$

$\frac{|4x-(-4)|}{2[x]+1}=\frac{+1}{0^+}=+\infty$

۵

$$\lim_{x\to 2^+}\frac{x^2-4}{x^3-[x^3]}$$

$x^3=8^- \to [x^3]=7$

$\frac{x^2-4}{x^3-7} \to \frac{x^2-4}{x^3-8}=\frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)}=\frac{4}{12}=\frac{1}{3}$

$$\lim_{x\to -1} \frac{x^2+10x+14}{12+4\sqrt[3]{x}} = \frac{4\cdot 5 - 10 + 14}{12-12} = \frac{0}{0}$$

$$\frac{x^2+10x+14}{4(2+\sqrt[3]{x})} \times \frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{\sqrt[3]{x^2}-2\sqrt[3]{x}+1} = \frac{(x+2)(x+7)(\sqrt[3]{x^2}-2\sqrt[3]{x}+1)}{4(x+8)} \xrightarrow{x\to -1} \frac{-4\times 12}{4} \quad (\times -12)$$

$-12$

۶

$$\lim \frac{\sqrt{2+\sqrt[3]{x}} - \sqrt{2-x}}{\sqrt{1-\cos x}} = \frac{\sqrt{2}-\sqrt{2}}{\sqrt{1-1}} = \frac{0}{0} \longrightarrow \text{(ضرب مزدوج)} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} \times \frac{\sqrt{2+\sqrt[3]{x}} + \sqrt{2-x}}{\sqrt{2+\sqrt[3]{x}} + \sqrt{2-x}} \Rightarrow$$

$$\frac{(2+\sqrt[3]{x}-2+x)(\sqrt{1+\cos x})}{(-\sin x)(\sqrt{2+\sqrt[3]{x}}+\sqrt{2-x})} \xrightarrow{x\to 0} \frac{-2\sqrt{2}}{-2\sqrt{2}} = (-2)$$

$$\sqrt{1-\cos^2 x} = \sqrt{\sin^2 x} = |\sin x| \left(\neq \sin\bar{x}\right) = \sin x$$

۷

$$\lim \frac{k+\cos\sqrt{a}\,x}{kx^2} = 2 \xrightarrow[\cos x = 1-\frac{x^2}{2}]{x\to 0} \frac{k+1-\frac{ax^2}{2}}{kx^2} = 2 \qquad k+1=0 \Rightarrow k=-1$$

$$k=-1 \rightsquigarrow \frac{-\frac{ax^2}{2}}{-x^2} = \frac{ax^2}{2x^2} = 2 \longrightarrow a=4$$

$$\frac{a}{k} = \frac{4}{-1} \quad (-4)$$

۸

$$\lim_{x\to (\sqrt[3]{a})^+} \frac{\sqrt[3]{x-\sqrt[3]{a}} + \sqrt[3]{x}\sqrt{x-\sqrt[3]{a}}}{\sqrt{x^2-ax^2}} = \frac{0}{0^+}$$

$$\frac{\sqrt{x-\sqrt[3]{a}}}{\sqrt{x-\sqrt[3]{a}}\sqrt{x+\sqrt[3]{a}}} + \frac{\sqrt[3]{x}-\sqrt[3]{a}}{\sqrt{x^2-ax^2}} \times \frac{\sqrt{x}+\sqrt[3]{a}}{\sqrt{x}+\sqrt[3]{a}} = \frac{1}{\sqrt{x+\sqrt[3]{a}}} + \frac{2(x-\sqrt[3]{a})}{\sqrt{x-\sqrt[3]{a}}\sqrt{x+\sqrt[3]{a}}(\sqrt{x}+\sqrt[3]{a})}$$

$$x\to \sqrt[3]{a} \longrightarrow \frac{1}{\sqrt{4a}} + 2\times 2 = \frac{1}{\sqrt{9a}} = \frac{2}{\sqrt{4a}}$$

۹

$$\lim_{x\to -1} \frac{1-k[x]}{x^2-1} = -\infty$$

$x\to (-1)^+ \longrightarrow [x]=-1 \longrightarrow \frac{1+k}{x^2-1} = \frac{1+k}{0^-} = -\infty \longrightarrow k>-1$

$x<(-1)^- \longrightarrow [x]=-2 \longrightarrow \frac{1+2k}{x^2-1} = \frac{1+2k}{0^+} = -\infty \longrightarrow k<-\frac{1}{2}$

$-1<k<-\frac{1}{2}$

$-\pi < k\pi < -\frac{\pi}{2}$


۱۰