

نام و نام خانوادگی پاسخنامه تشریحی تکلیف شماره ۵ ... کلاس ... ریاضی دوم دبیرستان

$\lim_{x \rightarrow 2} \frac{x^2 - a}{x^2 + ax + b} = -\infty \xrightarrow{\text{جایگذاری}} \lim_{x \rightarrow 2} \frac{-1}{x^2 + ax + b} = -\infty \rightarrow \text{باید مخرج همواره (0) باشد تا پاسخ } (-\infty) \text{ شود.}$

$\Rightarrow x^2 + ax + b = (x-2)^2 \Rightarrow x^2 + ax + b = x^2 - 4x + 4 \Rightarrow a = -4, b = 4 \quad / \quad a+b = -4+4=0$

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$\lim_{x \rightarrow \sqrt[3]{4}} \frac{x}{x^2 + ax + b} = +\infty \xrightarrow{\text{جایگذاری}} \lim_{x \rightarrow \sqrt[3]{4}} \frac{\sqrt[3]{4}}{x^2 + ax + b} = +\infty \rightarrow \text{همانند سوال قبل، مخرج باید همواره (0) شود.}$
 (دایره‌ای $x = \sqrt[3]{4}$ تا پاسخ حد $(+\infty)$ شود.)

$\Rightarrow x^2 + ax + b = (x - \sqrt[3]{4})^2 \Rightarrow x^2 + ax + b = x^2 - 2\sqrt[3]{4}x + (\sqrt[3]{4})^2 \Rightarrow a = -2\sqrt[3]{4}, b = (\sqrt[3]{4})^2$
 $\left[\frac{b}{a}\right] = \left[\frac{(\sqrt[3]{4})^2}{-2\sqrt[3]{4}}\right] = \left[-\frac{\sqrt[3]{4}}{2}\right] = \left[-\frac{2^{\frac{1}{3}}}{2}\right] = \left[-\frac{1}{2^{\frac{2}{3}}}\right] = \left[-\frac{1}{\sqrt[3]{4}}\right] = \boxed{-1}$

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$\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{[x] + a}{\left|\frac{\sqrt{x}}{x} + a\right| + a} = -\infty \xrightarrow{\text{مخرج به 0 میل کند}} \left|\frac{1}{\sqrt{2}} + a\right| + a = 0 \Rightarrow a = -\frac{1}{\sqrt{2}}, [a] = \boxed{-1}$

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$\lim_{x \rightarrow (-\frac{1}{2})^+} \frac{14x - [-\frac{1}{2} \times \frac{1}{x^2}]}{25x + [3 \times \frac{1}{x^2}]} \Rightarrow \lim_{x \rightarrow (-\frac{1}{2})^+} \frac{14x - [-\frac{1}{2} \times \frac{1}{x^2}]}{25x + [3 \times \frac{1}{x^2}]} \rightarrow \lim_{x \rightarrow (-\frac{1}{2})^+} \frac{14x + 1}{25x + 12} = \lim_{x \rightarrow (-\frac{1}{2})^+} \frac{14 \left(\frac{1}{2} + \epsilon\right)}{12 \left(\frac{1}{2} + \epsilon\right)} = \frac{1}{12}$

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$\frac{1}{2x} > 4 \rightarrow \frac{-1}{2x} < -8 \rightarrow \left[\frac{-1}{2x}\right] = -9$

$\lim_{x \rightarrow (-\frac{1}{2})^+} = \frac{14x + 1}{25x + 12} = \lim_{x \rightarrow (-\frac{1}{2})^+} \frac{-8 + 1}{0^+} = \frac{-7}{0^+} = -\infty$

$\lim_{x \rightarrow 2^+} \frac{x^2 - 4}{x^2 - [x^2]} \rightarrow \lim_{x \rightarrow 2^+} \frac{x^2 - 4}{x^2 - 1} = \lim_{x \rightarrow 2^+} \frac{(x-2)(x+2)}{(x-1)(x^2 + x + 1)} = \frac{4}{12} = \boxed{\frac{1}{3}}$

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$$\lim_{x \rightarrow -1} \frac{x^2 + 1 \cdot x + 1}{1 + 4\sqrt{x}} = \frac{0}{0} \quad \Rightarrow \quad \lim_{x \rightarrow -1} \frac{x^2 + 1 \cdot x + 1}{4(\sqrt{x} + \sqrt{x})} \times \frac{(\sqrt{x} + \sqrt{x})(\sqrt{x} - \sqrt{x})}{(\sqrt{x} + \sqrt{x})(\sqrt{x} - \sqrt{x})} = \lim_{x \rightarrow -1} \frac{(x+1)(x+1)(\sqrt{x} - \sqrt{x})}{4(1+x)} = \lim_{x \rightarrow -1} \frac{(x+1)(x+1)(\sqrt{x} - \sqrt{x})}{4(1+x)}$$

$$\Rightarrow \lim_{x \rightarrow -1} \frac{(x+1)(\sqrt{x})}{4} = \frac{-1}{4} \quad \checkmark$$

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$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - \sqrt{1-x}}{\sqrt{1-\cos x}} \xrightarrow{\text{KOP}} \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - \sqrt{1-x}}{\sqrt{\frac{x^2}{2}}} \xrightarrow{\text{KOP}} \lim_{x \rightarrow 0} \frac{\frac{1}{2\sqrt{x+1}} + \frac{1}{2\sqrt{1-x}}}{-\frac{x}{\sqrt{2}}}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\frac{1}{2\sqrt{x+1}} + \frac{1}{2\sqrt{1-x}}}{-\frac{x}{\sqrt{2}}} = \frac{-1}{\sqrt{2}} \quad \checkmark$$

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$$\lim_{x \rightarrow 0} \frac{K + \cos(\sqrt{a}x)}{Kx^2} \xrightarrow{\text{KOP}} \lim_{x \rightarrow 0} \frac{-\sqrt{a} \sin(\sqrt{a}x)}{2Kx} \xrightarrow{\text{KOP}} \lim_{x \rightarrow 0} \frac{-a \cos(\sqrt{a}x)}{2K} = \frac{-a}{2K}$$

$$\Rightarrow \frac{-a}{2K} = \frac{1}{2} \Rightarrow \frac{a}{K} = -1 \quad \checkmark$$

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$$\lim_{x \rightarrow \infty} \frac{\sqrt{x-a} + \sqrt{x+a}}{\sqrt{x-a} \sqrt{x+a}} \times \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}} = \lim_{x \rightarrow \infty} \frac{\sqrt{x-a} (\sqrt{x} + \sqrt{a}) + \sqrt{x+a} (\sqrt{x} + \sqrt{a})}{\sqrt{x-a} \sqrt{x+a} (\sqrt{x} + \sqrt{a})}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{\sqrt{x-a} (\sqrt{x} + \sqrt{a}) + \sqrt{x+a} (\sqrt{x} + \sqrt{a})}{\sqrt{x-a} \sqrt{x+a} (\sqrt{x} + \sqrt{a})} = \frac{\sqrt{a}}{\sqrt{a}} = \frac{1}{\sqrt{a}} \quad \checkmark$$

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$$\lim_{x \rightarrow -1} \frac{1 - K[x]}{x^2 - 1} = -\infty \quad \begin{cases} \lim_{x \rightarrow (-1)^+} \frac{1 - K[x]}{x^2 - 1} = \lim_{x \rightarrow (-1)^+} \frac{1 + K}{0^+} = -\infty \Rightarrow 1 + K < 0 \Rightarrow K < -1 \quad (1) \\ \lim_{x \rightarrow (-1)^-} \frac{1 - K[x]}{x^2 - 1} = \lim_{x \rightarrow (-1)^-} \frac{1 + K}{0^+} = -\infty \Rightarrow 1 + K < 0 \Rightarrow K < -1 \quad (2) \end{cases}$$

$$\textcircled{1} \textcircled{2} \Rightarrow -1 < K < -\frac{1}{2} \quad \rightarrow \quad -\pi < K\pi < -\frac{\pi}{2} \quad \rightarrow \quad \frac{1}{2} < K < 1 \quad \rightarrow \quad \frac{1}{2} < K < 1$$

طول عرض نقاب را به سمت راست
در نتیجه نامشروع

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