

$$\lim_{n \rightarrow \infty} \frac{r_n - a}{n^r + an + b} = \frac{-r}{\infty} = 0 \rightarrow \frac{-r}{(n-r)^r} = \frac{-r}{\frac{n^r - r n^r + r^2}{a + b}} = 0 \quad (1)$$

$a + b = 0$ ✓

$$\lim_{n \rightarrow \sqrt[r]{r}} \frac{n}{n^r + an + b} = \frac{\sqrt[r]{r}}{(n - \sqrt[r]{r})^r} = +\infty \quad (2)$$

$$\frac{\sqrt[r]{r}}{n^r - r\sqrt[r]{r} + \sqrt[r]{r^2}} = +\infty \quad \left[\frac{b}{a} \right] = \left[\frac{\sqrt[r]{14}}{-\sqrt[r]{32}} \right] = \left[\frac{-1}{\sqrt[r]{2}} \right] = -1 \quad (3)$$

$$\lim_{n \rightarrow (\frac{1}{\sqrt{w}})^+} \left[\frac{\sqrt{r}}{n} + a \right] + a = -\infty \rightarrow \left[-\frac{1}{\sqrt{w}} \right] + a = -\infty$$

$$\frac{-1 + a}{\left| \frac{1}{\sqrt{w}} + a \right| + a} = -\infty \rightarrow \begin{cases} \text{num} < 0 \rightarrow a < 1 \\ \text{den} = 0^+ \rightarrow \frac{1}{\sqrt{w}} + a + a = 0 \rightarrow a = -\frac{1}{4} \end{cases}$$

برای مخرج مثبت و صورت منفی، مخرج باید مثبت و صورت منفی باشد

$\left[-\frac{1}{4} \right] = -1$ ✓

$$\lim_{n \rightarrow (-\frac{1}{r})^+} \frac{14n - \left[-\frac{r}{n^r} \right]}{r^r n + \left[\frac{r}{n^r} \right]} = \frac{-14 + 9}{-1r^r + 1r^r} = \frac{1}{0^+} = +\infty \quad (4)$$

$n > -\frac{1}{r} \rightarrow n^r < \frac{1}{r} \rightarrow \frac{1}{n^r} > r \rightarrow \frac{r}{n^r} < -1$

$$\lim_{n \rightarrow r^+} \frac{n^r - r}{n^r - [n^r]} = \frac{n^r - r}{n^r - r} = \frac{(n-r)(n+r)}{(n-r)(n^r + n + r)} = \frac{r}{1r} = \frac{1}{r} \quad (5)$$

$$\lim_{n \rightarrow -1} \frac{n^2 + 1 \cdot n + 1}{1 + 4\sqrt{n}} = \frac{(n+1)(n+2)}{4(1+\sqrt{n})} = \frac{(n+1)(n+2)(1)}{4(1+\sqrt{n})} \quad \text{L'Hôpital's Rule}$$

$$= \frac{-4 \times 1}{4} = -1 \quad \checkmark$$

$$\lim_{n \rightarrow 0} \frac{\sqrt{1+\sqrt{n}} - \sqrt{1-\sqrt{n}}}{\sqrt{1-\cos n}} = \frac{(1+\sqrt{n}) - (1-\sqrt{n})}{2\sqrt{2} \sqrt{\frac{n}{2}}} = \frac{2\sqrt{n}}{2\sqrt{2} \times \frac{\sqrt{n}}{\sqrt{2}}} \quad \text{L'Hôpital's Rule}$$

$$= \frac{2\sqrt{n}}{2\sqrt{n}} = 1 \quad \checkmark$$

$$\lim_{n \rightarrow 0} \frac{k + \cos(\sqrt{a}n)}{kn^2} = \frac{f'}{g'} = \frac{-\sqrt{a} \sin(\sqrt{a}n)}{2kn} = \frac{0}{0} \quad \text{L'Hôpital's Rule}$$

$$f'' = \frac{-a \cos(\sqrt{a}n)}{2k} = \frac{-a}{2k} = \frac{a}{k} = -4 \quad \checkmark$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n^2 - 9a^2} + \sqrt{n^2} - \sqrt{n^2 + 9a^2}}{\sqrt{n^2 - 9a^2}} = \frac{0}{0} \quad \text{L'Hôpital's Rule}$$

$$\frac{\sqrt{n^2 - 9a^2} + \sqrt{n^2} - \sqrt{n^2 + 9a^2}}{\sqrt{n^2 - 9a^2}} = \frac{\sqrt{n^2 - 9a^2} + \sqrt{n^2} - \sqrt{n^2 + 9a^2}}{\sqrt{n^2 - 9a^2}} + \frac{\sqrt{n^2} - \sqrt{n^2 + 9a^2}}{\sqrt{n^2 - 9a^2}}$$

$$\rightarrow \frac{1}{\sqrt{4a}} + 0 = \frac{1}{\sqrt{4a}} \quad \checkmark$$

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$$\lim_{n \rightarrow -1} \frac{1 - K[n]}{n^2 - 1} = -\infty$$

$(-1)^+$ $\frac{1+K}{0^-} \rightarrow -\infty \rightarrow K > -1$
 $(-1)^-$ $\frac{1+2K}{0^+} \rightarrow -\infty \rightarrow K < -\frac{1}{2}$

(1.)

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$$\rightarrow -1 \left\{ K < -\frac{1}{2} \right\} \rightarrow -\pi < K\pi < -\frac{\pi}{2} \rightarrow \text{Pole}$$

