مبینا رضایی

۱)
$$\lim_{x\to 2}\frac{2x-5}{x^2+ax+b}=\frac{-4}{x^2+ax+b}=-\infty \longrightarrow \frac{-4}{(x-2)^2}=\frac{-4}{x^2\underbrace{-4}_{a}x\underbrace{+4}_{b}}=-\infty$$

$a+b=0$

۲)
$$\lim_{x\to\sqrt[3]{4}}\frac{x}{x^2+ax+b}=+\infty \longrightarrow \frac{\sqrt[3]{4}}{(x-\sqrt[3]{4})^2}=+\infty$$

$$\longrightarrow \frac{\sqrt[3]{4}}{x^2-2\sqrt[3]{4}+\sqrt[3]{16}}=+\infty \qquad \left[\frac{b}{a}\right]=\left[\frac{\sqrt[3]{16}}{-\sqrt[3]{32}}\right]=\left[-\frac{1}{\sqrt[3]{2}}\right]=-1$$

۳)
$$\lim_{x\to\left(\frac{1}{\sqrt{3}}\right)^+}\frac{[-x]+a}{\left|\frac{\sqrt{3}}{3}x+a\right|+a}=-\infty \longrightarrow \frac{\overbrace{\left[-\frac{1}{\sqrt{3}}\right]}^{-1}+a}{\left|\frac{1}{3}+a\right|+a}=-\infty$$

$$\frac{-1+a}{\left|\frac{1}{3}+a\right|+a}=-\infty$$

→ صورت $<0 \longrightarrow a<1$

→ مخرج $=0^+ \longrightarrow \frac{1}{3}+a+a=0 \longrightarrow a=-\frac{1}{6}$

اگر قدرمطلق را منفی برداریم مخرج $\frac{1}{3}$ می‌شود نه صفر $\qquad \left[-\frac{1}{6}\right]=-1$

۴)
$$\lim_{x\to\left(-\frac{1}{2}\right)^+}\frac{16x-\left[-\frac{2}{x^2}\right]}{24x+\left[\frac{3}{x^2}\right]}=\frac{-8+9}{-12^++12}=\frac{1}{0^+}=+\infty$$

$$x>-\frac{1}{2}\rightarrow x^2<\frac{1}{4}\rightarrow \frac{1}{x^2}>4 \rightarrow -\frac{2}{x^2}<-8$$

۵)
$$\lim_{x\to 2^+}\frac{x^2-4}{x^3-[x^3]}=\frac{x^2-4}{x^3-8}=\frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)}=\frac{4}{12}=\frac{1}{3}$$

۶-

$$\lim_{x\to -8} \frac{x^2+10x+16}{12+6\sqrt[3]{x}} = \frac{(x+8)(x+2)}{6(2+\sqrt[3]{x})} = \frac{(x+8)(x+2)(12)}{6(\cancel{8+x})}$$

$$= \frac{-6\times 12}{6} = -12$$

(۷

$$\lim_{x\to 0} \frac{\sqrt{2+3x}-\sqrt{2-x}}{\underbrace{\sqrt{1-\cos x}}_{\text{هم‌ارزی}}} = \frac{(2+3x)-(2-x)}{2\sqrt{2}\,\sqrt{\frac{x^2}{2}}} = \frac{4x}{2\sqrt{2}\times\frac{-x}{\sqrt{2}}}$$

$$= \frac{4x}{-2x} = -2$$

(۸

$$\lim_{x\to 0} \frac{k+\cos(\sqrt{a}\,x)}{kx^2} = 3 \xrightarrow{f'} \frac{-\sqrt{a}\sin(\sqrt{a}\,x)}{2kx} = 3$$

$$\xrightarrow{f''} \frac{-a\cos(\sqrt{a}\,x)}{2k} = \frac{-a}{2k} = 3 \longrightarrow \boxed{\frac{a}{k} = -6}$$

(۹

$$\lim_{x\to (3a)^+} \frac{2\sqrt{x-3a}+3\sqrt{x}-3\sqrt{3a}}{\underbrace{\sqrt{x^2-9a^2}}_{(\sqrt{x-3a})(\sqrt{x+3a})}} = \frac{0}{0}$$

$$\longrightarrow \frac{2\sqrt{x-3a}}{(\sqrt{x-3a})(\sqrt{x+3a})} + \frac{3(\sqrt{x}-\sqrt{3a})}{(\sqrt{x-3a})(\sqrt{x+3a})} = \frac{2}{\sqrt{x+3a}} + \frac{3\cancel{(x-3a)}}{\cancel{(\sqrt{x-3a})}(\sqrt{x+3a})\times 2\sqrt{3a}}$$

$$\sqrt{x-3a} \to 0$$

$$\longrightarrow \frac{2}{\sqrt{6a}} + 0 = \frac{2}{\sqrt{6a}}$$

(۱.

$$\lim_{x \to -1} \frac{1 - K[x]}{x^2 - 1} = -\infty \quad \begin{cases} \xrightarrow{(-1)^+} \dfrac{1 + K}{0^-} = -\infty \to K > -1 \\[2ex] \xrightarrow{(-1)^-} \dfrac{1 + 2K}{0^+} = -\infty \to K < -\dfrac{1}{2} \end{cases}$$

$$\longrightarrow -1 < K < -\frac{1}{2} \longrightarrow -\pi < K\pi < -\frac{\pi}{2} \longrightarrow$$ ناحیه سوم