دوازدهم رشته ریاضی — مبینا هاتف الحسینی

۱۲/۵

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$a+b=?$ $\quad x=2$

$$\lim_{x\to 2}\frac{\overset{-1}{\overbrace{2x-5}}}{\underbrace{x^2+ax+b}_{0^+}}=-\infty$$

$x=2$ ریشه مضاعف مخرج است پس:

$(x-2)^2=x^2-4x+4 \Rightarrow \begin{cases} a=-4 \\ b=4\end{cases} \boxed{a+b=\text{صفر}}$

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$$\lim_{x\to(\sqrt[3]{4})}\frac{\overset{L>0}{\overbrace{x}}}{\underbrace{x^2+ax+b}_{0^+}}=+\infty \Rightarrow \left[\frac{b}{a}\right]=?$$

$x=\sqrt[3]{4}$

$(x-\sqrt[3]{4})^2=x^2-2\sqrt[3]{4}\,x+\sqrt[3]{4^2}$ $\qquad \sqrt[3]{2^4}=2\sqrt[3]{2}$

$x^2\underbrace{-2\sqrt[3]{4}}_{a}\,x+\underbrace{2\sqrt[3]{2}}_{b} \Rightarrow \left[\frac{b}{a}\right]=\left[\frac{-\cancel{2}\sqrt[3]{2}}{\cancel{2}\sqrt[3]{4}}\right]$

$\boxed{\left[-\sqrt[3]{\frac{2}{4}}\right]=-1}$

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$$\lim_{x\to(\frac{1}{\sqrt{3}})^+}\frac{[-x]+a}{\left|\frac{\sqrt{3}}{3}x+a\right|+a}=-\infty \Rightarrow [a]=?$$

$$\lim_{x\to(\frac{1}{\sqrt{3}})^+}\frac{\overset{-1}{\cancel{\left[-\frac{1}{\sqrt{3}}\right]}}+a}{\left|\frac{\cancel{\sqrt{3}}}{3}\times\frac{1}{\cancel{\sqrt{3}}}+a\right|+a}=-\infty \Rightarrow \frac{a-1}{\left|\frac{1}{3}+a\right|+a}=-\infty$$

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$$-\infty\begin{cases}\frac{L>0}{0^-} \to a-1>0 \Rightarrow a>1 \quad \frac{1}{3}+2a=0 \Rightarrow a=-\frac{1}{6}\ \times \\ \frac{L<0}{0^+} \to a-1<0 \Rightarrow a<1 \quad \frac{1}{3}+2a=0 \Rightarrow a=-\frac{1}{6}\ \checkmark\end{cases}$$

$\boxed{[a]=\left[-\frac{1}{6}\right]=-1}$

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$$\lim_{x\to 0^-} \frac{\sqrt{2+3x}-\sqrt{2-x}\times 4}{\sqrt{1-\cos x}} = \frac{0}{0}$$

$$\sqrt{2\sin^2\frac{x}{2}} = \sqrt{2}\,\left|\sin\frac{x}{2}\right| = -\sqrt{2}\sin\frac{x}{2}$$

$$\lim_{x\to 0^-} \frac{\cancel{2}+3x-\cancel{2}+x\ (4)}{(2\sqrt{2})(-\sqrt{2}\sin\frac{x}{2})} = \lim_{x\to 0^-} \frac{4x}{-2\sqrt{2}\ \cancel{\sin}\frac{x}{2}}$$

$$\lim_{x\to 0^-} \frac{4x}{-\cancel{2}\sqrt{2}\times\frac{x}{\cancel{2}}} = \frac{-4}{\sqrt{2}} = \frac{-4\sqrt{2}}{2} = \boxed{-2\sqrt{2}}$$

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$$\lim_{x\to 0} \frac{k+\cos(\sqrt{a}\,x)}{kx^2} = 3 \Rightarrow \frac{a}{k} = ?$$

با قرار دادن صفر به جای x حاصل به صورت $\frac{k+1}{0}$ می‌شود پس حاصل حد به بی‌نهایت میل کند اما چون حاصل حد یک عدد شده پس صورت صفر می‌شود:

$$k+1=0$$
$$k=-1$$

$$\lim_{x\to 0} \frac{\cos(\sqrt{a}\,x)-1}{-x^2} = \frac{0}{0} \xrightarrow{hop} \lim_{x\to 0} \frac{\cancel{-}\sqrt{a}\ \cancel{\sin}x}{\cancel{-}2x} = 3$$

$$\Rightarrow \sqrt{a}=6 \Rightarrow a=36 \Rightarrow \frac{a}{k} = \frac{36}{-1} = \boxed{-36}$$

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$$\lim_{x\to (3a)^+} \frac{2\sqrt{x-3a}+3\sqrt{x}-\overset{3\sqrt{3a}}{\cancel{\sqrt{27a}}}}{\sqrt{x^2-9a^2}} = \frac{0}{0} \quad (a>0)$$

$$\sqrt{x^2-9a^2} \to (\sqrt{x-3a})(\sqrt{x+3a})$$

$$\frac{2}{\sqrt{x+3a}} + \frac{3(\sqrt{x}-\sqrt{3a})}{\sqrt{x-3a}\,\sqrt{x+3a}} \sim \frac{2}{\sqrt{x+3a}} + \frac{3(\cancel{x-3a})(\sqrt{x-3a})}{\cancel{\sqrt{x-3a}}\,\sqrt{x+3a}\,(2\sqrt{3a})}$$

$$\frac{\overset{2}{\cancel{4}}\sqrt{3a}+3\cancel{\sqrt{x-3a}}^{\,0}}{(\sqrt{x+3a})(\cancel{2}\sqrt{3a})} = \frac{2}{\sqrt{6a}} = \frac{2\sqrt{6a}}{3\cancel{6}a} = \frac{\sqrt{6a}}{3}$$

$$\lim_{x\to 0^-} \frac{\sqrt{2+3x}-\sqrt{2-x}\times 2}{\sqrt{1-\cos x}} = \frac{0}{0}$$ ۷-

$$\sqrt{2\sin^2\frac{x}{2}} = \sqrt{2}\left|\sin\frac{x}{2}\right| = -\sqrt{2}\sin\frac{x}{2}$$

$$\lim_{x\to 0^-} \frac{\cancel{2}+3x-\cancel{2}+x\,(\cancel{2})}{(2\sqrt{2})(-\cancel{\sqrt{2}}\sin\frac{x}{2})} = \lim_{x\to 0^-} \frac{\cancel{4x}\quad 4\sqrt{2}\,x}{-2\sqrt{2}\,\cancel{\sin}\frac{x}{2}}$$ ۱٫۵

$$\lim_{x\to 0^-} \frac{4\cancel{x}}{-\cancel{2}\sqrt{2}\times\frac{\cancel{x}}{\cancel{2}}} = \frac{-4}{\sqrt{2}} = \frac{-4\sqrt{2}}{2} = -\cancel{2\sqrt{2}}\quad -2$$

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$$\lim_{x\to 0} \frac{k+\cos(\sqrt{a}\,x)}{kx^2} = 3 \Rightarrow \frac{a}{k} = ?$$ ۱

با قرار دادن صفر به جای x حاصل به صورت $\frac{k+1}{0}$ می‌شود پس حاصل حد به بی‌نهایت میل کند اما چون حاصل حد یک عدد شده پس صورت صفر می‌شود

$k+1=0$

$k=-1$

$$\lim_{x\to 0} \frac{-1+\cos(\sqrt{a}\,x)}{-x^2} \xrightarrow{\text{هم ارزی}} \lim_{x\to 0} \frac{-1+1-\frac{(\sqrt{a}\,x)^2}{2}}{-x^2} = \lim_{x\to 0} \frac{\frac{-ax^2}{2}}{-x^2} = \frac{a}{2} \to \frac{a}{2} = 3 \to a = 6$$

$$\lim_{x\to 0} \frac{\cos(\sqrt{a}\,x)-1}{-x^2} = \frac{0}{0} \xrightarrow{hop} \lim_{x\to 0} \frac{\cancel{1}\sqrt{a}\,\cancel{\sin x}}{\cancel{+}2\cancel{x}} = 3$$

$$\frac{a}{k} = \frac{6}{-1} = -6$$

$$\Rightarrow \sqrt{a} = 6 \Rightarrow a = 36 \Rightarrow \frac{a}{k} = \frac{36}{-1} = -36$$

$$\lim_{x\to(3a)^+} \frac{2\sqrt{x-3a}+3\sqrt{x}-\overset{3\sqrt{3a}}{\cancel{\sqrt{27a}}}}{\sqrt{x^2-9a^2}\to(\sqrt{x-3a})(\sqrt{x+3a})} = \frac{0}{0}\quad (a>0)$$ ۹-

$$\frac{2}{\sqrt{x+3a}} + \frac{3(\sqrt{x}-\sqrt{3a})}{\sqrt{x-3a}\,\sqrt{x+3a}} \sim \frac{2}{\sqrt{x+3a}} + \frac{3(\cancel{x-3a})(\sqrt{x-3a})}{\cancel{\sqrt{x-3a}}\,\sqrt{x+3a}\,(2\sqrt{3a})}$$

$$\frac{2\,\cancel{2\sqrt{3a}}+3\sqrt{\cancel{x-3a}}^{\,0}}{(\sqrt{x+3a})(\cancel{2\sqrt{3a}})} = \frac{2}{\sqrt{6a}} = \frac{2\sqrt{6a}}{36a} = \frac{\sqrt{6a}}{3}$$ ۲

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$$\lim_{x \to (-1)} \frac{1 - k[x]}{x^2 - 1} = -\infty \qquad (k\pi , \cos k\pi)$$

در کدام ناحیه؟

$$\lim_{x \to (-1)^+} \frac{1 + k}{x^2 - 1} = -\infty \qquad 1 + k > 0 \Rightarrow k > -1 \qquad 1^- - 1 = 0^-$$

$$\lim_{x \to (-1)^-} \frac{1 + 2k}{x^2 - 1} = -\infty \qquad 1 + 2k < 0 \Rightarrow k < \frac{-1}{2} \qquad 1^+ - 1 = 0^+$$

$$-1 < k < -\frac{1}{2}$$

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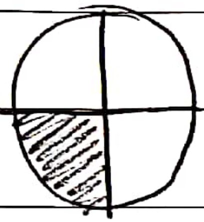
$$-\pi < k\pi < -\frac{\pi}{2} \qquad \cos(-\pi) < \cos k\pi < \cos\left(-\frac{\pi}{2}\right)$$

$$-1 < \cos k\pi < 0$$



$\Rightarrow$ ناحیه سوم

سوال ۴

$$x > -\frac{1}{2} \to x^2 < \frac{1}{4} \to \frac{1}{x^2} > 4 \to \frac{3}{x^2} > 12 \to \left[\frac{3}{x^2}\right] = 12$$

$$\frac{1}{x^2} > 4 \to \frac{-2}{x^2} < -8 \to \left[\frac{-2}{x^2}\right] = -9$$

$$\lim_{x \to (-\frac{1}{2})^+} = \frac{16x+9}{24x+12} = \lim_{x \to (-\frac{1}{2})^+} \frac{-8+9}{0^+} = \frac{1}{0^+} = +\infty$$

سوال ۵

$$\lim_{x \to 2^+} \frac{x^2-4}{x^3-[x^3]} = \lim_{x \to 2^+} \frac{x^2-4}{x^3-8} \to \lim_{x \to 2^+} \frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)} = \lim_{x \to 2^+} \frac{x+2}{x^2+2x+4} = \frac{4}{12} = \frac{1}{3}$$

سوال ۶

$$\lim_{x \to -8} \frac{x^2+10x+16}{12+6\sqrt[3]{x}} = \lim_{x \to -8} \frac{(x+8)(x+2)}{6(2+\sqrt[3]{x})} \times \frac{\sqrt[3]{x^2}+4-2\sqrt[3]{x}}{\sqrt[3]{x^2}+4-2\sqrt[3]{x}} \to$$

$$\lim_{x \to -8} \frac{(x+8)(x+2)\times 12}{6(x+8)} \to \lim_{x \to -8} \frac{12(x+2)}{6} = 2\times(-6) = -12$$