۱ـ

$$\lim_{x\to 2}\frac{\overset{-1}{\overline{2x-5}}}{x^2+ax+b_{\,0^+}}=-\infty \qquad a+b=? \qquad x=2$$

در ریشه مضاعف مخرج است پس:

$$(x-2)^2=x^2-4x+4 \Rightarrow \begin{cases} a=-4 \\ b=4 \end{cases} \quad \boxed{a+b=\text{صفر}}$$

۲ـ

$$\lim_{x\to(\sqrt[3]{4})}\frac{\overset{L>0}{x}}{x^2+ax+b_{\to 0^+}}=+\infty \Rightarrow \left[\frac{b}{a}\right]=?$$

$x=\sqrt[3]{4}$

$$(x-\sqrt[3]{4})^2=x^2-2\sqrt[3]{4}\,x+\sqrt[3]{4^2}$$

$$\sqrt[3]{2^4}=2\sqrt[3]{2}$$

$$x^2\underbrace{-2\sqrt[3]{4}}_{a}\,x+\underbrace{2\sqrt[3]{2}}_{b} \Rightarrow \left[\frac{b}{a}\right]=\left[\frac{-\cancel{2}\sqrt[3]{2}}{\cancel{2}\sqrt[3]{4}}\right]$$

$$\boxed{\left[-\sqrt[3]{\frac{2}{4}}\right]=-1}$$

۳ـ

$$\lim_{x\to\left(\frac{1}{\sqrt{3}}\right)^+}\frac{[-x]+a}{\left|\frac{\sqrt{3}}{3}x+a\right|+a}=-\infty \Rightarrow [a]=?$$

$$\lim_{x\to\left(\frac{1}{\sqrt{3}}\right)^+}\frac{\cancel{\left[-\frac{1}{\sqrt{3}}\right]}^{\,-1}+a}{\left|\frac{\cancel{\sqrt{3}}}{3}\times\frac{1}{\cancel{\sqrt{3}}}+a\right|+a}=-\infty \Rightarrow \frac{a-1}{\left|\frac{1}{3}+a\right|+a}=-\infty$$

$$-\infty\begin{cases}\dfrac{L>0}{0^-}\to a-1>0\Rightarrow a>1 \qquad \dfrac{1}{3}+2a=0\Rightarrow a=\dfrac{-1}{6}\ \times \\[2ex] \dfrac{L<0}{0^+}\to a-1<0\Rightarrow a<1 \qquad \dfrac{1}{3}+2a=0\Rightarrow a=\dfrac{-1}{6}\ \checkmark\end{cases}$$

$$\boxed{[a]=\left[-\frac{1}{6}\right]=-1}$$

$$\lim_{x\to 0^-} \frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}} \times \dots = \frac{0}{0}$$ -۷

$$\sqrt{2\sin^2\frac{x}{2}} = \sqrt{2}\left|\sin\frac{x}{2}\right| = -\sqrt{2}\sin\frac{x}{2}$$

$$\lim_{x\to 0^-} \frac{\cancel{2}+3x-\cancel{2}+x\ (\cancel{\sqrt{2}})}{(2\sqrt{2})(-\cancel{\sqrt{2}}\sin\frac{x}{2})} = \lim_{x\to 0^-} \frac{4x}{-2\sqrt{2}\ \cancel{\sin}\frac{x}{2}}$$

$$\lim_{x\to 0^-} \frac{4\cancel{x}}{-\cancel{2}\sqrt{2}\times\frac{\cancel{x}}{\cancel{2}}} = \frac{-4}{\sqrt{2}} = \frac{-4\sqrt{2}}{2} = \boxed{-2\sqrt{2}}$$

-۸

$$\lim_{x\to 0} \frac{k+\cos(\sqrt{a}\,x)}{kx^2} = 3 \Rightarrow \frac{a}{k} = ?$$

با قرار دادن صفر به جای x حاصل به صورت $\frac{k+1}{0}$ می‌شود پس حاصل حد به بی‌نهایت میل کند اما چون حاصل حد یک عدد شده پس صورت صفر می‌شود:

$k+1=0$

$k=-1$

$$\lim_{x\to 0} \frac{\cos(\sqrt{a}\,x)-1}{-x^2} = \frac{0}{0} \xrightarrow{hop} \lim_{x\to 0} \frac{\cancel{-}\sqrt{a}\,\cancel{\sin x}}{\cancel{-}2x} = 3$$

$$\Rightarrow \sqrt{a} = 6 \Rightarrow a = 36 \Rightarrow \frac{a}{k} = \frac{36}{-1} = \boxed{-36}$$

$$\lim_{x\to (3a)^+} \frac{2\sqrt{x-3a}+3\sqrt{x}-\overset{3\sqrt{3a}}{\cancel{\sqrt{27a}}}}{\sqrt{x^2-9a^2} \to (\sqrt{x-3a})(\sqrt{x+3a})} = \frac{0}{0} \quad (a>0)$$ -۹

$$\frac{2}{\sqrt{x+3a}} + \frac{3(\sqrt{x}-\sqrt{3a})}{\sqrt{x-3a}\,\sqrt{x+3a}} \sim \frac{2}{\sqrt{x+3a}} + \frac{3(\cancel{x-3a})(\sqrt{x-3a})}{\cancel{\sqrt{x-3a}}\,\sqrt{x+3a}\,(2\sqrt{3a})}$$

$$\frac{2\cancel{\times 2}\sqrt{3a}+3\cancel{\sqrt{x-3a}}^{\,0}}{(\sqrt{x+3a})(\cancel{2}\cancel{\sqrt{3a}})} = \frac{2}{\sqrt{6a}} = \frac{\cancel{2}\sqrt{6a}}{3\cancel{6}a} = \frac{\sqrt{6a}}{3}$$

۷-

$$\lim_{x\to 0^-}\frac{\sqrt{2+3x}-\sqrt{2-x}\times 4}{\sqrt{1-\cos x}}=\frac{0}{0}$$

$$\sqrt{2\sin^2\frac{x}{2}}=\sqrt{2}\left|\sin\frac{x}{2}\right|=-\sqrt{2}\sin\frac{x}{2}$$

$$\lim_{x\to 0^-}\frac{\cancel{2}+3x-\cancel{2}+x\,(\cancel{4})}{(2\sqrt{2})(-\cancel{\sqrt{2}}\sin\frac{x}{2})}=\lim_{x\to 0^-}\frac{4x}{-2\sqrt{2}\,\cancel{\sin}\frac{x}{2}}$$

$$\lim_{x\to 0^-}\frac{4\cancel{x}}{-\cancel{2}\sqrt{2}\times\frac{\cancel{x}}{\cancel{2}}}=\frac{-4}{\sqrt{2}}=\frac{-4\sqrt{2}}{2}=-2\sqrt{2}$$

۸-

$$\lim_{x\to 0}\frac{k+\cos(\sqrt{a}\,x)}{kx^2}=3\implies\frac{a}{k}=?$$

با قرار دادن صفر به جای x حاصل به صورت $\frac{k+1}{0}$ در می‌آید پس حاصل حد به بی‌نهایت میل کند اما چون حاصل حد یک عدد شده پس صورت صفر می‌شود:

$$k+1=0$$
$$k=-1$$

$$\lim_{x\to 0}\frac{\cos(\sqrt{a}\,x)-1}{-x^2}=\frac{0}{0}\xrightarrow{hop}\lim_{x\to 0}\frac{\cancel{-}\sqrt{a}\,\cancel{\sin x}}{\cancel{-}2\cancel{x}}=3$$

$$\implies\sqrt{a}=6\implies a=36\implies\frac{a}{k}=\frac{36}{-1}=-36$$

۹-

$$\lim_{x\to(3a)^+}\frac{2\sqrt{x-3a}+3\sqrt{x}-\cancel{\sqrt{27a}}^{\,3\sqrt{3a}}}{\sqrt{x^2-9a^2}}=\frac{0}{0}\quad(a>0)$$

$$\sqrt{x^2-9a^2}\to(\sqrt{x-3a})(\sqrt{x+3a})$$

$$\frac{2}{\sqrt{x+3a}}+\frac{3(\sqrt{x}-\sqrt{3a})}{\sqrt{x-3a}\,\sqrt{x+3a}}\sim\frac{2}{\sqrt{x+3a}}+\frac{3(\cancel{x-3a})(\sqrt{x-3a})}{\cancel{\sqrt{x-3a}}\,\sqrt{x+3a}\,(2\sqrt{3a})}$$

$$\frac{\cancel{4}^{\,2}\sqrt{3a}+3\cancel{\sqrt{x-3a}}^{\,0}}{(\sqrt{x+3a})(\cancel{4\sqrt{3a}})}=\frac{2}{\sqrt{6a}}=\frac{2\sqrt{6a}}{3\cancel{6}a}=\frac{\sqrt{6a}}{3}$$

۱۰.

$(k\pi , \cos k\pi)$ در کدام ناحیه؟

$$\lim_{x \to (-1)} \frac{1 - k[x]}{x^2 - 1} = -\infty$$

$$\lim_{x \to (-1)^+} \frac{1 + k}{x^2 - 1} = -\infty$$

$1 + k > 0 \Rightarrow k > -1$

$1^- - 1 = 0^-$

$$\lim_{x \to (-1)^-} \frac{1 + 2k}{x^2 - 1} = -\infty$$

$1 + 2k < 0 \Rightarrow k < \frac{-1}{2}$

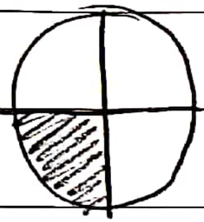
$1^+ - 1 = 0^+$

$$\Big\{ \; -1 < k < -\frac{1}{2}$$

$$-\pi < k\pi < -\frac{\pi}{2}$$

$$\cos(-\pi) < \cos k\pi < \cos\left(-\frac{\pi}{2}\right)$$

$$-1 < \cos k\pi < 0$$



$\Rightarrow$ ناحیه سوم