

$$\lim_{x \rightarrow r} \frac{x^r - r^r}{x^r + ax + b} = -\infty$$

$$\Rightarrow x^r + ax + b = (x - r)^r$$

$$x^r - \varepsilon x + \varepsilon = x^r + ax + b \rightarrow a = -\varepsilon, b = \varepsilon$$

$$a + b = 0$$

$$\lim_{x \rightarrow \sqrt[r]{\varepsilon}} \frac{x^r - \sqrt[r]{\varepsilon}^r}{x^r + ax + b} = +\infty$$

$$\Rightarrow x^r + ax + b = (x - \sqrt[r]{\varepsilon})^r$$

$$x^r - \sqrt[r]{\varepsilon}^r x + \sqrt[r]{\varepsilon}^r = x^r + ax + b \rightarrow a = -\sqrt[r]{\varepsilon}^r, b = \sqrt[r]{\varepsilon}^r$$

$$\left[\frac{b}{a}\right] = \left[\frac{\sqrt[r]{\varepsilon}^r}{-\sqrt[r]{\varepsilon}^r}\right] = \left[-\sqrt[r]{\frac{1}{\varepsilon}}\right] = (-1)$$

$$\lim_{x \rightarrow (\frac{1}{\sqrt[r]{a}})^+} \frac{[-x] + a}{|\sqrt[r]{\frac{1}{x}} + a| + a} = -\infty \rightarrow \lim_{x \rightarrow (\frac{1}{\sqrt[r]{a}})^+} \frac{a - 1}{|\frac{1}{x} + a| + a} = -\infty$$

$$|\frac{1}{x} + a| = -a \rightarrow a < 0 \rightarrow \frac{1}{x} + a = -a \quad a = -\frac{1}{4}$$

$$[a] = \left[-\frac{1}{4}\right] = (-1)$$

$$\lim_{x \rightarrow (-\frac{1}{r})^+} \frac{14x - \left[-\frac{r}{x^r}\right]}{r\varepsilon x + \left[\frac{r}{x^r}\right]}$$

$$x > -\frac{1}{r} \rightarrow x^r > \frac{1}{\varepsilon} \rightarrow \frac{1}{x^r} < \varepsilon \rightarrow \left[-\frac{r}{x^r}\right] = -1$$

$$x > -\frac{1}{r} \rightarrow x^r > \frac{1}{\varepsilon} \rightarrow \frac{1}{x^r} < \varepsilon \rightarrow \left[\frac{r}{x^r}\right] = 11$$

$$x > -\frac{1}{r} \rightarrow x^r < \frac{1}{\varepsilon} \rightarrow \frac{1}{x^r} > \varepsilon \rightarrow \left[\frac{r}{x^r}\right] = 12$$

$$= \lim_{x \rightarrow (-\frac{1}{r})^+} \frac{14x + 1}{r\varepsilon x + 11} = 0$$

$$\frac{1}{x^r} > \varepsilon \rightarrow \left[-\frac{r}{x^r}\right] = -9$$

$$\lim_{x \rightarrow (-\frac{1}{r})^+} \frac{14x + 9}{r\varepsilon x + 12} = \lim_{x \rightarrow (-\frac{1}{r})^+} \frac{-1 + 9}{0^+} = \frac{1}{0^+} = +\infty$$

$$\lim_{x \rightarrow r^+} \frac{x^r - \varepsilon}{x^r - [x^r]} = \lim_{x \rightarrow r^+} \frac{x^r - \varepsilon}{x^r - 1} = \lim_{x \rightarrow r^+} \frac{(x - r)(x + r)}{(x - r)(x^r + rx + \varepsilon)} = \frac{\varepsilon}{12} = \left(\frac{1}{12}\right)$$

$$\lim_{x \rightarrow -1} \frac{x^2 + 10x + 14}{1x + 4\sqrt[3]{x}} \xrightarrow{\text{hop}} \lim_{x \rightarrow -1} \frac{x^2 + 10x + 14}{\sqrt[3]{x^3}} = \lim_{x \rightarrow -1} (x+0)(\sqrt[3]{x^3})$$

$$= -3 \times 2 = -6 \quad \checkmark$$

(2)

6

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1-\cos x}} \times \frac{\text{uof}}{\text{uof}} \times \frac{\text{LT}}{\text{LT}} = \lim_{x \rightarrow 0^-} \frac{(1+x) - (1-x)}{|\sin x| \times 2\sqrt{x}}$$

$$\lim_{x \rightarrow 0^-} \frac{\varepsilon x}{-2 \sin x} = \frac{\varepsilon x}{-2x} = -\frac{1}{2} \quad \checkmark$$

(2)

7

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$$\lim_{x \rightarrow 0} k + \cos(\sqrt{x}) = 0 \rightarrow k+1=0 \rightarrow k=-1$$

$$\lim_{x \rightarrow 0} \frac{-1 + \cos(\sqrt{x})}{-x^2} \xrightarrow{\text{هم‌انز}}$$

$$\frac{a}{k} = \frac{4}{-1} = -4$$

8

$$\lim_{x \rightarrow \frac{1}{2}^+} \frac{x\sqrt{x}-\frac{1}{2}}{\sqrt{x}-\frac{1}{2}\sqrt{x+\frac{1}{2}}} + \frac{x\sqrt{x}-\frac{1}{2}\sqrt{x}}{\sqrt{x}-\frac{1}{2}\sqrt{x+\frac{1}{2}}} = \lim_{x \rightarrow \frac{1}{2}^+} \left(\frac{x}{\sqrt{x}} + \frac{x(\sqrt{x}-\sqrt{x+\frac{1}{2}})}{\sqrt{x}-\frac{1}{2}\sqrt{x+\frac{1}{2}}} \times \frac{\sqrt{x+\frac{1}{2}}}{\sqrt{x+\frac{1}{2}}} \right) \rightarrow$$

$$\lim_{x \rightarrow \frac{1}{2}^+} \left(\frac{x}{\sqrt{x}} + \frac{x(x-\frac{1}{2})}{(\sqrt{x}-\frac{1}{2}\sqrt{x+\frac{1}{2}})(\sqrt{x+\frac{1}{2}})} \right) = \lim_{x \rightarrow \frac{1}{2}^+} \left(\frac{x}{\sqrt{x}} + \frac{x\sqrt{x}-\frac{1}{2}}{(\sqrt{x}-\frac{1}{2}\sqrt{x+\frac{1}{2}})(\sqrt{x+\frac{1}{2}})} \right) = \lim_{x \rightarrow \frac{1}{2}^+} \frac{x}{\sqrt{x}} + 0$$

$$\rightarrow \frac{\sqrt{x}}{\sqrt{x}} = \sqrt{\frac{x}{x}} = \sqrt{\frac{1}{2}}$$

$$\lim_{x \rightarrow -1} \frac{1 - k[x]}{x^2 - 1} = -\infty \rightarrow -1^+ : \frac{1+k}{0^-} = -\infty \quad 1+k > 0 \rightarrow k > -1$$

$$\rightarrow -1^- : \frac{1+k}{0^+} = -\infty \quad 1+k < 0 \rightarrow k < -1$$

$$-1 < k < -\frac{1}{2} \rightarrow -1 < \cos k\pi < 0 \rightarrow \frac{1}{2} \approx 0.5 \quad \checkmark$$

(2)

10