حل «سرفصل» «تکلیف شماره ۲۰» «دوازدهم دفتر ۸»

۱۴٫۵

① $\lim_{x\to 2} \frac{\sqrt{2x-5}}{x^2+ax+b} = -\infty$ ، $a+b=-4+4=0$

$2^+, 2^- \to$ منفی

$x^2+ax+b \to (x-2)^2 = x^2-4x+4$

(۲)

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② $\lim_{x\to \sqrt[3]{4}} \frac{x}{x^2+ax+b} = +\infty$

$\to (x-\sqrt[3]{4})^2 = 0 \to x^2 \underbrace{-2\sqrt[3]{4}}_{a}\,x + \underbrace{\sqrt[3]{16}}_{b}$

$\left[\frac{b}{a}\right] \to \left[\frac{\sqrt[3]{16}}{-2\sqrt[3]{4}}\right] = \left[\frac{\sqrt[3]{4}}{-2}\right] \to \left[-\frac{1}{\sqrt[3]{2}}\right] = \cancel{2}$ -1

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③ $\lim_{x\to (\frac{1}{\sqrt{3}})^+} \frac{[-x]+a}{\left|\frac{\sqrt{3}}{3}x+a\right|+a} = -\infty \to \left[\left(-\frac{1}{\sqrt{3}}\right)^+\right] \to -1$

$\to \frac{1}{3}+a+a=0 \to \frac{1}{3}+2a=0 \to \boxed{a=-\frac{1}{6}} \to \left[-\frac{1}{6}\right] = -1$

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④ $\lim_{x\to (-\frac{1}{2})^+} \frac{16x-\left[-\frac{2}{x^2}\right]}{24x+\left[\frac{3}{x^2}\right]} \to \frac{16x+8}{24x+12} \to \frac{8(2x+1)}{12(2x+1)} = \frac{2}{3}$

$x>-\frac{1}{2} \to \frac{1}{x}<-2 \to \frac{1}{x^2}>4 \to -\frac{2}{x^2}<-8$

$x>-\frac{1}{2} \to \frac{1}{x}<-2 \to \frac{1}{x^2}>4 \to \frac{3}{x^2}>12$

$\frac{1}{x^2}>4 \to \frac{-2}{x^2}<-8 \to \left[\frac{-2}{x^2}\right]=-9$

$\lim_{x\to(-\frac{1}{2})^+} \frac{16x+9}{24x+12} = \lim_{x\to(-\frac{1}{2})^+} \frac{-8+9}{0^+} = \frac{1}{0^+} = +\infty$

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⑤ $\lim_{x\to 2^+} \frac{x^2-4}{x^3-[x^3]} \to \frac{x^2-4}{x^3-8} \to \frac{\cancel{(x-2)}\overset{4}{(x+2)}}{\cancel{(x-2)}\underbrace{(x^2+2x+4)}_{12}} = \frac{1}{3}$

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⑥ $\lim_{x\to -8} \frac{x^2+10x+16}{12+6\sqrt[3]{x}} \to \frac{(x+2)(x+8)\times(\sqrt[3]{x^2}-2\sqrt[3]{x}+4)}{6(2+\sqrt[3]{x})\times(\sqrt[3]{x^2}-2\sqrt[3]{x}+4)}$

$\to \frac{(x+2)\cancel{(x+8)}(\sqrt[3]{x^2}-2\sqrt[3]{x}+4)}{6\cancel{(x+8)}} = -12$

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7) $$\lim_{x\to 0^-} \frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}$$

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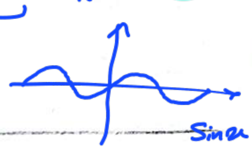
$$\lim_{x\to 0^-} \frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{2+3x}+\sqrt{2-x}}{\sqrt{2+3x}+\sqrt{2-x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} \rightarrow$$

$$\lim_{x\to 0^-} \frac{2+3x-2+x}{\sqrt{1-\cos^2 x}} \times \frac{\sqrt{1+\cos x}}{2\sqrt{2}} = \lim_{x\to 0^-} \frac{4x\times\sqrt{2}}{|\sin x|\times 2\sqrt{2}} =$$

$$\frac{(\sqrt{2+3x}-\sqrt{2-x})(\sqrt{1+\cos x})}{\sqrt{1-\cos^2}\ \sin x} \longrightarrow -(\sqrt{2+3x} -$$

$$\lim_{x\to 0^-} \frac{4x\times\sqrt{2}}{-2\sqrt{2}\sin x} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0^-} \underbrace{\frac{x}{\sin x}}_{1} \times \frac{4\sqrt{2}}{-2\sqrt{2}} = -2$$

$$\left(1+\frac{1}{2}\cancel{\cos x}\right)$$



8) $$\lim_{x\to 0} \frac{k+\cos(\sqrt{a}\,x)}{kx^2} = 3 \longrightarrow \frac{k+1-\frac{ax^2}{2}}{kx^2} = 3$$

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$$\rightarrow \frac{2k+2-ax^2}{2kx^2} = 3 \longrightarrow \begin{cases} 2k+2=0 \rightarrow 2k=-2 \rightarrow k=-1 \\ \dfrac{-ax^2}{-2x^2} = 3 \rightarrow a=6 \end{cases} \longrightarrow \frac{a}{k} = -6$$

9) $$\lim_{x\to (3a)^+} \frac{2\sqrt{x-3a}+3\sqrt{x}-\sqrt{27a}}{\sqrt{x^2-9a^2}}$$

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$$\lim_{x\to 3a^+} \frac{2\sqrt{x-3a}}{\sqrt{x-3a}\sqrt{x+3a}} + \frac{3\sqrt{x}-3\sqrt{3a}}{\sqrt{x-3a}\sqrt{x+3a}} = \lim_{x\to 3a^+}\left(\frac{2}{\sqrt{6a}} + \frac{3(\sqrt{x}-\sqrt{3a})}{\sqrt{x-3a}\sqrt{x+3a}} \times \frac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}+\sqrt{3a}}\right) \rightarrow$$

$$\lim_{x\to 3a^+}\left(\frac{2}{\sqrt{6a}} + \frac{3(x-3a)}{(\sqrt{x-3a}\sqrt{x+3a})(\sqrt{x}+\sqrt{3a})}\right) = \lim_{x\to 3a^+}\left(\frac{2}{\sqrt{6a}} + \frac{3\sqrt{x-3a}\to 0}{(\sqrt{x+3a})(\sqrt{x}+\sqrt{3a})}\right) = \lim_{x\to 3a^+} \frac{2}{\sqrt{6a}} + 0$$

$$\frac{\sqrt{4}}{\sqrt{6a}} = \sqrt{\frac{4}{6a}} = \sqrt{\frac{2}{3a}}$$

10) $$\lim_{x\to -1} \frac{1-k[x]}{x^2-1} = -\infty \xrightarrow{-1^+} \frac{1+k}{x^2-1} = -\infty \Rightarrow \frac{1+k}{0^-} \longrightarrow k>-1$$

$$\xrightarrow{-1^-} \frac{1+2k}{x^2-1} = -\infty \longrightarrow \frac{1+2k}{0^+} \longrightarrow 1+2k<0 \longrightarrow k<-\frac{1}{2}$$

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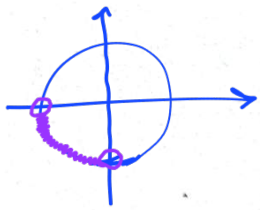
$$\longrightarrow -1<k<-\frac{1}{2} \longrightarrow$$

$k\pi$ نقطه → ربع سوم

$\cos k\pi$ نقطه → ربع اول

$$-1<k<-\frac{1}{2} \rightarrow \begin{cases} -\pi<k\pi<-\frac{\pi}{2} \\ -1<\cos k\pi<0 \end{cases}$$



ناحیه سوم