تشریح کامل «سرفصل» «تکلیف شماره ۲۰» «دوازدهم دفتر ۲»

۱) $\lim\limits_{x\to 2} \dfrac{\overbrace{\sqrt[3]{2x-5}}^{2^+,2^- \to \text{منفی}}}{\underbrace{x^2+ax+b}} = -\infty \quad , \quad a+b=-4+4=0$

$\hookrightarrow (x-2)^2 = x^2-4x+4$

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۲) $\lim\limits_{x\to \sqrt[3]{4}} \dfrac{x}{x^2+ax+b} = +\infty$

$\left[\dfrac{b}{a}\right] \to \left[\dfrac{\sqrt[3]{16}}{2\sqrt[3]{4}}\right] = \left[2^{\frac{1}{3}}\right] \to \left[\dfrac{1}{\sqrt[3]{2}}\right] = 0$

$\to (x-\sqrt[3]{4})^2 = 0 \to x^2 \underbrace{-2\sqrt[3]{4}}_{a}\,x + \underbrace{\sqrt[3]{16}}_{b}$

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۳) $\lim\limits_{x\to \left(\frac{1}{\sqrt{3}}\right)^+} \dfrac{[-x]+a}{\left|\frac{\sqrt{3}}{3}x+a\right|+a} = -\infty \to \left[\left(-\dfrac{1}{\sqrt{3}}\right)^+\right] = -1$

$\to \dfrac{1}{3}+a+a=0 \to \dfrac{1}{3}+2a=0 \to \boxed{a=-\dfrac{1}{6}} \to \left[-\dfrac{1}{6}\right] = -1$

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۴) $\lim\limits_{x\to \left(-\frac{1}{2}\right)^+} \dfrac{16x-\left[-\frac{2}{x^2}\right]}{24x+\left[\frac{3}{x^2}\right]} \to \dfrac{16x+8}{24x+12} \to \dfrac{8(2x+1)}{12(2x+1)} = \dfrac{2}{3}$

$x > -\dfrac{1}{2} \to \dfrac{1}{x} < -2 \to \dfrac{1}{x^2} > 4 \to -\dfrac{2}{x^2} < -8$

$x > -\dfrac{1}{2} \to \dfrac{1}{x} < -2 \to \dfrac{1}{x^2} > 4 \to \dfrac{3}{x^2} > 12$

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۵) $\lim\limits_{x\to 2^+} \dfrac{x^2-4}{x^3-[x^3]} \to \dfrac{x^2-4}{x^3-8} \to \dfrac{\cancel{(x-2)}\overbrace{(x+2)}^{4}}{\cancel{(x-2)}\underbrace{(x^2+2x+4)}_{12}} = \dfrac{1}{3}$

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۶) $\lim\limits_{x\to -8} \dfrac{x^2+10x+16}{12+6\sqrt[3]{x}} \to \dfrac{(x+2)(x+8)\times(\sqrt[3]{x^2}-2\sqrt[3]{x}+4)}{6(2+\sqrt[3]{x})\times(\sqrt[3]{x^2}-2\sqrt[3]{x}+4)}$

$\to \dfrac{\overset{-6}{(x+2)}\cancel{(x+8)}\,(\sqrt[3]{\overset{4}{x^2}}-2\sqrt[3]{\overset{-2}{x}}+4)}{6\cancel{(x+8)}} = -12$

۷) $\lim\limits_{x\to 0^-} \dfrac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}} \times \dfrac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}$

$\dfrac{(\sqrt{2+3x}-\sqrt{2-x})(\sqrt{1+\cos x})}{\sqrt{\cancel{1-\cos^2}}\ |\sin x|} \to -(\sqrt{2+3x} - \dots$

$\cancel{1+\frac{1}{2}\cos x}$

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۸) $\lim\limits_{x\to 0} \dfrac{k+\cos(\sqrt{a}x)}{kx^2} = 3 \Rightarrow \dfrac{k+1-\frac{ax^2}{2}}{kx^2} = 3$

$\to \dfrac{2k+2-ax^2}{2kx^2} = 3 \to \begin{cases} 2k+2=0 \to 2k=-2 \to k=-1 \\ \dfrac{-ax^2}{-2x^2} = 3 \to a=6 \end{cases} \to \dfrac{a}{k} = -6$

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۹) $\lim\limits_{x\to (3a)^+} \dfrac{2\sqrt{x-3a}+3\sqrt{x}-\sqrt{27a}}{\sqrt{x^2-9a^2}}$

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۱۰) $\lim\limits_{x\to -1} \dfrac{1-k[x]}{x^2-1} = -\infty \xrightarrow{-1^+} \dfrac{1+k}{x^2-1} = -\infty \Rightarrow \dfrac{1+k}{0^-} \to k>-1$

$\xrightarrow{-1^-} \dfrac{1+2k}{x^2-1} = -\infty \to \dfrac{1+2k}{0^+} \to 1+2k<0 \to k<-\frac{1}{2}$

$\to -1<k<-\frac{1}{2} \to$ نقطه $k\pi$ ↓ در ناحیه سوم — نقطه $\cos k\pi$ ↓ در ناحیه اول