تکلیف شماره (۲۰)

برنیان شمشیری - دوازدهم دفتر A ۱۱

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$$\lim_{x\to 2}\frac{2x-5}{x^2+ax+b}=-\infty \begin{cases} 2^+: \lim \frac{-1}{4+2a+b}=-\infty \to x^2+ax+b=0^+ \ (x\to2^+) \\ 2^-: \lim \frac{-1}{4+2a+b}=-\infty \to x^2+ax+b=0^+ \ (x\to 2^-) \end{cases}$$

$\to (x-2)^2 = x^2-4x+4 = x^2+ax+b$

$\to a=-4 \ , \ b=4 \quad \Rightarrow a+b=-4+4=0$ ✓

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$$\lim_{x\to\sqrt[3]{4}}\frac{x}{x^2+ax+b}=+\infty \begin{cases} \frac{2^{\frac23}}{0^+}=+\infty \\ \frac{2^{\frac23}}{0^+}=+\infty \end{cases}$$

$\Rightarrow$ مخرج: $(x-2^{\frac23})^2 = x^2+2^{\frac43}-2^{\frac53}x = x^2+ax+b$

$\to a=-2^{\frac53} \ , \ b=2^{\frac43}$

$\to [\frac{b}{a}] = [\frac{2^{\frac43}}{-2^{\frac53}}] = [-2^{-\frac13}] = [\frac{1}{\sqrt[3]{-2}}] = -1$ ✓

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$$\lim_{x\to(\frac{1}{\sqrt3})^+}\frac{[-x]+a}{|\frac{\sqrt3}{3}x+a|+a}=-\infty \Rightarrow \lim\frac{[-\frac{\sqrt3}{3}]+a}{|\frac13+a|+a}=-\infty \Rightarrow \lim\frac{-1+a}{\ }$$

حاصل حد به $-\infty$ میل کرده ← مخرج به صفر میل می‌کند.

$\lim_{x\to(\frac{1}{\sqrt3})^+} = |\frac{\sqrt3}{3}x+a| = |\frac{1}{\sqrt3}\times\frac{\sqrt3}{3}+a|+a=0$

$\to |\frac13+a|+a=0 \to \begin{cases} a\geq-\frac13 \to 2a+\frac13=0 \to a=-\frac16 \ ✓ \\ a<-\frac13 \to -a-\frac13+a=0 \ ✗ \end{cases}$

$\to [a]=[-\frac16]=-1$

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$$\lim_{x\to(\frac{-1}{2})^+}\frac{16x-[\frac{-2}{x^2}]}{24x+[\frac{3}{x^2}]} = \frac{(-8)^+-[(-8)^-]}{(-12)^++[12^+]} = \frac{(-8)^+-(-9)}{(-12)^++12} = \frac{1}{0^+} = +\infty$$

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* $x>-\frac12 \to 16x>-8$
* $x>-\frac12 \to x^2<\frac14 \to \frac{2}{x^2}>8 \to \frac{-2}{x^2}<-8$
* $x>-\frac12 \to 24x>-12$
* $x>-\frac12 \to x^2<\frac14 \to \frac{3}{x^2}>12$

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$$\lim_{x\to2^+}\frac{x^2-4}{x^3-[x^3]} = \lim_{x\to2^+}\frac{x^2-4}{x^3-[8^+]} = \lim_{x\to2^+}\frac{x^2-4}{x^3-8} = \frac{(\cancel{x-2})(x+2)}{(\cancel{x-2})(x^2+4+2x)} = \frac{4}{4+4+4} = \frac{4}{12} = \frac13$$ ✓

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$$\lim_{x\to-8}\frac{x^2+10x+16}{12+6\sqrt[3]{x}} = \frac{64-80+16}{12-12} = \frac00 \xrightarrow{\text{رفع ابهام}} \lim_{x\to-8}\frac{x^2+10x+16}{12+6\sqrt[3]{x}} = \frac{(x+8)(x+2)}{6(2+\sqrt[3]{x})}\times\frac{(4+\sqrt[3]{x^2}-2\sqrt[3]{x})}{(4+\sqrt[3]{x^2}-2\sqrt[3]{x})}$$

$$= \frac{(\cancel{x+8})(x+2)(4+\sqrt[3]{x^2}-2\sqrt[3]{x})}{6(\cancel{8+x})} = \frac{(x+2)(4+\sqrt[3]{x^2}-2\sqrt[3]{x})}{6} = \frac{(-6)(4+4+4)}{6} = \frac{-6\times12}{6} = \boxed{-12}$$ ✓

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$$\lim_{x\to0^-}\frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}} = \frac{\sqrt2-\sqrt2}{\sqrt{1-1}} = \frac00 \xrightarrow{\text{رفع ابهام}} \lim\frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}}\times\frac{\sqrt{2+3x}+\sqrt{2-x}}{\sqrt{2+3x}+\sqrt{2-x}} = \frac{(2+3x)-(2-x)}{\sqrt{1-\cos x}\cdot(\sqrt{2+3x}+\sqrt{2-x})} = \frac{4x}{\cancel{x}\,2\sqrt2}$$

$$= \frac{4}{2\sqrt2} = \frac{2}{\sqrt2} = \frac{2\sqrt2}{2} = \sqrt2$$

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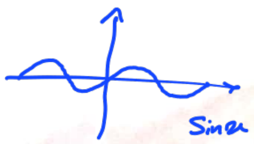
$$\lim_{x\to0^-}\frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}}\times\frac{\sqrt{2+3x}+\sqrt{2-x}}{\sqrt{2+3x}+\sqrt{2-x}}\times\frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} \to \lim_{x\to0^-}\frac{2+3x-2+x}{\sqrt{1-\cos^2x}}\times\frac{\sqrt{1+\cos x}}{2\sqrt2} =$$

$$\lim_{x\to0^-}\frac{4x\times\sqrt2}{|\sin x|\times2\sqrt2} = \lim_{x\to0^-}\frac{4x\sqrt2}{-2\sqrt2\sin x} \xrightarrow{\text{فشردگی}} \lim_{x\to0^-}\underbrace{\frac{x}{\sin x}}_{1}\times\frac{4\sqrt2}{-2\sqrt2} = -2$$



سوال ۸ — حد مخرج صفر و حاصل حد عددی حقیقی و غیر صفر ← حد صورت نیز برابر صفر بوده

$$\lim_{x\to 0} K+\cos(\sqrt{a}\,x)=0 \to K+1=0 \to K=-1$$

$$\lim_{x\to 0}\frac{-1+\cos(\sqrt{a}\,x)}{-x^2}\xrightarrow{\text{هم ارزی}}\lim_{x\to 0}\frac{-1+1-\frac{(\sqrt{a}\,x)^2}{2}}{-x^2}=\lim_{x\to 0}\frac{\frac{-ax^2}{2}}{-x^2}=\frac{a}{2}\to\frac{a}{2}=3\to a=6$$

$$\frac{a}{K}=\frac{6}{-1}=-6$$

سوال ۹

$$\lim_{x\to 3a^+}\frac{2\sqrt{x-3a}}{\sqrt{x-3a}\,\sqrt{x+3a}}+\frac{3\sqrt{x}-3\sqrt{3a}}{\sqrt{x-3a}\,\sqrt{x+3a}}=\lim_{x\to 3a^+}\left(\frac{2}{\sqrt{6a}}+\frac{3(\sqrt{x}-\sqrt{3a})}{\sqrt{x-3a}\,\sqrt{x+3a}}\times\frac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}+\sqrt{3a}}\right)\to$$

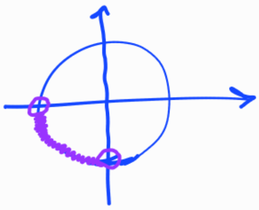
$$\lim_{x\to 3a^+}\left(\frac{2}{\sqrt{6a}}+\frac{2(x-3a)}{(\sqrt{x-3a}\,\sqrt{x+3a})(\sqrt{x}+\sqrt{3a})}\right)=\lim_{x\to 3a^+}\left(\frac{2}{\sqrt{6a}}+\frac{3\overbrace{\sqrt{x-3a}}^{\to 0}}{(\sqrt{x}+\sqrt{3a})(\sqrt{x+3a})}\right)=\lim_{x\to 3a^+}\frac{2}{\sqrt{6a}}+0$$

$$\to\frac{\sqrt{4}}{\sqrt{6a}}=\sqrt{\frac{4}{6a}}=\sqrt{\frac{2}{3a}}$$

سوال ۱۰

$$\begin{cases}\lim\limits_{x\to(-1)^+}\dfrac{1-K[x]}{x^2-1}=\lim\limits_{x\to(-1)^+}\dfrac{1-K(-1)}{0^-}=\dfrac{1+K}{0^-}=-\infty\to 1+K>0\to K>-1 & (\mathrm{I})\\[2ex] \lim\limits_{x\to(-1)^-}\dfrac{1-K[x]}{x^2-1}=\lim\limits_{x\to(-1)^-}\dfrac{1-K(-2)}{0^+}=\dfrac{1+2K}{0^+}=-\infty\to 1+2K<0\to K<-\dfrac{1}{2} & (\mathrm{II})\end{cases}$$

$$(\mathrm{I})\cap(\mathrm{II})\longrightarrow -1<K<-\frac{1}{2}\to\begin{cases}-\pi<K\pi<-\dfrac{\pi}{2}\\ -1<\cos K\pi<0\end{cases}$$



ناحیه سوم