

(۱)

$$\lim_{x \rightarrow r} \frac{x^n - a}{x^r + a} = -\infty \Rightarrow \lim_{x \rightarrow r} \frac{-1}{x^r + a} = -\infty \Rightarrow \lim_{x \rightarrow r} \frac{-1}{(x-r)^r}$$

$$= \lim_{x \rightarrow r} \frac{-1}{x^r - r^n + r} = -\infty \Rightarrow a = -r, b = r \Rightarrow a + b = 0 \quad \checkmark$$

(۲)

$$\lim_{x \rightarrow \sqrt[r]{r}} \frac{x}{x^r + a} = +\infty \Rightarrow \lim_{x \rightarrow \sqrt[r]{r}} \frac{\sqrt[r]{r}}{(x - \sqrt[r]{r})^r} = +\infty$$

$$\Rightarrow \lim_{x \rightarrow \sqrt[r]{r}} \frac{\sqrt[r]{r}}{x^r - r\sqrt[r]{r} + \sqrt[r]{r}} = +\infty \Rightarrow \left[\frac{b}{a} \right] = \left[\frac{\sqrt[r]{14}}{-r\sqrt[r]{r}} \right] = \left[\frac{\sqrt[r]{14}}{-\sqrt[r]{r^r}} \right]$$

(۳)

$$= \left[\frac{-1}{\sqrt[r]{r}} \right] = -1 \quad \checkmark$$

(۴)

$$\lim_{x \rightarrow (\frac{1}{\sqrt{p}})^+} \frac{[-x] + a}{|\frac{\sqrt{p}}{p}x + a| + a} = -\infty \Rightarrow \frac{[-\frac{1}{\sqrt{p}}] + a}{|\frac{1}{p} + a| + a} = -\infty \Rightarrow \frac{(-1) + a}{|\frac{1}{p} + a| + a} = -\infty$$

(۵)

$$\Rightarrow \begin{cases} -1 + a < 0 \rightarrow a < 1 \\ |\frac{1}{p} + a| + a = 0^+ \Rightarrow \frac{1}{p} + a + a = 0 \Rightarrow 2a = -\frac{1}{p} \Rightarrow a = -\frac{1}{2p} \end{cases} \Rightarrow [a] = \left[-\frac{1}{2p} \right] = -1 \quad \checkmark$$

برای حل این مسئله، باید از شرط $a < 1$ و $2a = -\frac{1}{p}$ استفاده کرد. از $2a = -\frac{1}{p}$ داریم $a = -\frac{1}{2p}$. این مقدار را در $a < 1$ قرار می‌دهیم: $-\frac{1}{2p} < 1 \Rightarrow -1 < 2p \Rightarrow p > -\frac{1}{2}$. این شرط را در نظر می‌گیریم.

(۶)

$$\lim_{x \rightarrow (-\frac{1}{p})^+} \frac{14x - \left[-\frac{r}{x^r} \right]}{x^r + \left[\frac{r}{x^r} \right]} = \frac{-14 + \left[(-1)^r \right]}{-1^r + 1^r} = \frac{-14 + 9}{-1^r + 1^r} = \frac{-5}{0^+} = -\infty \quad \checkmark$$

$$x > -\frac{1}{p} \Rightarrow x^r < \frac{1}{p^r} \Rightarrow \frac{1}{x^r} > p^r \Rightarrow \frac{r}{x^r} > 1^r, \quad -\frac{r}{x^r} < -1$$

(۷)

$$\lim_{x \rightarrow 2^+} \frac{x^2 - 4}{x^3 - [x^3]} = \frac{x^2 - 4}{x^3 - 8} = \frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)} = \frac{4}{12} = \frac{1}{3} \quad \checkmark \quad (5)$$

$$\lim_{x \rightarrow -1} \frac{x^2 + \log x + 4}{12 + 9\sqrt{x}} = \frac{(x+1)(x+4)}{9(1+\sqrt{x})} \xrightarrow{\text{فيلڙج}} \frac{(x+1)(x+4)(3\sqrt{x})}{9(1+x)} \quad (6)$$

$$= \frac{(-4) \times 12}{9} = (-16) \quad \checkmark \quad (7)$$

$$\lim_{x \rightarrow 0^+} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1-\cos x}} \xrightarrow{\text{ساري}} \frac{(1+x^2) - (1-x^2)}{(1\sqrt{1})\sqrt{\frac{x^2}{2}}} = \frac{4x^2}{(1\sqrt{1})(\frac{x^2}{\sqrt{2}})} \quad (7)$$

$$= \frac{4x^2}{x^2} = 4 \quad \checkmark \quad (8)$$

$$\lim_{x \rightarrow 0} \frac{k + \cos(\sqrt{a}x)}{kx^2} \xrightarrow{\text{Hop}} \frac{-\sqrt{a} \sin(\sqrt{a}x)}{2kx} = \frac{0}{0} \quad (8)$$

$$\xrightarrow{\text{Hop}} \frac{-a \cos(\sqrt{a}x)}{2k} = \frac{-a}{2k} = \frac{a}{k} = -4 \quad \checkmark \quad (9)$$

$$\lim_{x \rightarrow (\sqrt{a})^+} \frac{\sqrt{x-a} + \sqrt{x-a} - \sqrt{x-a}}{\sqrt{x-a}} = \frac{0}{0} \quad \sqrt{x-a} = (\sqrt{x-a})(\sqrt{x+a}) \quad (9)$$

$$\Rightarrow \frac{\sqrt{x-a}}{(\sqrt{x-a})(\sqrt{x+a})} + \frac{\sqrt{x-a}}{(\sqrt{x-a})(\sqrt{x+a})} = \frac{1}{\sqrt{x+a}} + \frac{1}{\sqrt{x+a}} = \frac{2}{\sqrt{x+a}} \quad (10)$$

$$\Rightarrow \frac{2}{\sqrt{4a}} + 0 = \frac{2}{\sqrt{4a}} \quad \checkmark \quad (10)$$

$$\lim_{x \rightarrow (-1)} \frac{1-k[x]}{x^2-1} = -\infty \quad \begin{matrix} x \rightarrow (-1)^+ \\ \frac{1+k}{0^-} = -\infty \Rightarrow 1+k > 0 \Rightarrow k > -1 \end{matrix} \quad (10)$$

$$\quad \begin{matrix} x \rightarrow (-1)^- \\ \frac{1+k}{0^+} = -\infty \Rightarrow 1+k < 0 \Rightarrow k < -1 \end{matrix}$$

$$\Rightarrow -1 < k < -\frac{1}{p} \Rightarrow -\pi < k\pi < -\frac{\pi}{p}, \quad -1 < \cos k\pi < 0$$

(10)

در ناحیه سوم