

(1)

$$\lim_{x \rightarrow r} \frac{x^n - a}{x^p + ax + b} = -\infty \Rightarrow \lim_{x \rightarrow r} \frac{-1}{x^p + ax + b} = -\infty \Rightarrow \lim_{x \rightarrow r} \frac{-1}{(x-r)^p}$$

$$= \lim_{x \rightarrow r} \frac{-1}{x^p - rx + r} = -\infty \Rightarrow a = -r, b = r \Rightarrow \boxed{a+b=0}$$

$$\lim_{x \rightarrow \sqrt[3]{r}} \frac{x}{x^3 + ax + b} = +\infty \Rightarrow \lim_{x \rightarrow \sqrt[3]{r}} \frac{\sqrt[3]{r}}{(x - \sqrt[3]{r})^p} = +\infty$$

$$\Rightarrow \lim_{x \rightarrow \sqrt[3]{r}} \frac{\sqrt[3]{r}}{x^3 - r\sqrt[3]{r} + \sqrt[3]{14r}} = +\infty \Rightarrow \left[\frac{b}{a} \right] = \left[\frac{\sqrt[3]{14r}}{-r\sqrt[3]{r}} \right] = \left[\frac{\sqrt[3]{14r}}{-\sqrt[3]{r^4}} \right]$$

$$= \left[\frac{-1}{\sqrt[3]{r}} \right] = -1$$

(2)

$$\lim_{x \rightarrow (\frac{1}{\sqrt{p}})^+} \frac{[-x] + a}{|\frac{\sqrt{p}}{p}x + a| + a} = -\infty \Rightarrow \frac{[-\frac{1}{\sqrt{p}}] + a}{|\frac{1}{p} + a| + a} = -\infty \Rightarrow \frac{(-1) + a}{|\frac{1}{p} + a| + a} = -\infty$$

$$\Rightarrow \begin{cases} -1 + a < 0 \rightarrow a < 1 \\ |\frac{1}{p} + a| + a = 0^+ \Rightarrow \frac{1}{p} + a + a = 0 \Rightarrow 2a = -\frac{1}{p} \Rightarrow a = -\frac{1}{2p} \end{cases} \Rightarrow [a] = \left[-\frac{1}{2p} \right] = -1$$

برای منفی بودن عبارت خارج کسره می‌سازیم و در صورتی ساده می‌کنیم تا به جواب برسیم

(3)

$$\lim_{x \rightarrow (-\frac{1}{p})^+} \frac{14x - \left[-\frac{r}{x^2} \right]}{x^2 + \left[\frac{r}{x^2} \right]} = \frac{-1 + \left[(-1) \right]}{-1^2 + 1^2} = \frac{-1 + 9}{-1^2 + 1^2} = \frac{8}{0^+} = +\infty$$

$$x > -\frac{1}{p} \Rightarrow x^2 < \frac{1}{p^2} \Rightarrow \frac{1}{x^2} > p^2 \Rightarrow \frac{r}{x^2} > 1^2, \frac{-r}{x^2} < -1$$

$$\lim_{x \rightarrow 2^+} \frac{x^2 - 4}{x^3 - [x^3]} = \frac{x^2 - 4}{x^3 - 8} = \frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)} = \frac{4}{12} = \frac{1}{3} \quad (5)$$

$$\lim_{x \rightarrow -1} \frac{x^2 + \log x + 14}{12 + 9\sqrt{x}} = \frac{(x+1)(x+14)}{9(1+\sqrt{x})} \xrightarrow{\text{قيد كنج}} \frac{(x+1)(x+14)(3\sqrt{x})}{9(1+x)} \quad (6)$$

$$= \frac{(-4) \times 12}{9} = \frac{-48}{9} = \frac{-16}{3}$$

$$\lim_{x \rightarrow 0^+} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1-\cos x}} \xrightarrow{\text{قيد كنج}} \frac{(1+x^2) - (1-x^2)}{(1+\sqrt{1-x^2})\sqrt{\frac{x^2}{2}}} = \frac{2x^2}{(1+\sqrt{1-x^2})\sqrt{\frac{x^2}{2}}} \quad (7)$$

$$= \frac{2x}{1+\sqrt{1-x^2}} = \frac{2}{2} = 1$$

$$\lim_{x \rightarrow 0} \frac{k + \cos(\sqrt{a}x)}{kx^2} \xrightarrow{\text{Hop}} \frac{-\sqrt{a} \sin(\sqrt{a}x)}{2kx} = \frac{-\sqrt{a}}{2k} \quad (8)$$

$$\text{Hop} \rightarrow \frac{-a \cos(\sqrt{a}x)}{2k} = \frac{-a}{2k} = \frac{a}{k} = -4$$

$$\lim_{x \rightarrow (\sqrt{a})^+} \frac{\sqrt{x^2 - a^2} + \sqrt{x^2 - 9a^2} - \sqrt{x^2 - 4a^2}}{\sqrt{x^2 - 9a^2}} = \frac{a}{a} \quad \sqrt{x^2 - 9a^2} = (\sqrt{x^2 - a^2})(\sqrt{x^2 - 8a^2}) \quad (9)$$

$$\Rightarrow \frac{\sqrt{x^2 - a^2}}{(\sqrt{x^2 - a^2})(\sqrt{x^2 - 8a^2})} + \frac{\sqrt{x^2 - 8a^2}}{(\sqrt{x^2 - a^2})(\sqrt{x^2 - 8a^2})} - \frac{\sqrt{x^2 - 4a^2}}{(\sqrt{x^2 - a^2})(\sqrt{x^2 - 8a^2})} = \frac{1}{\sqrt{x^2 - 8a^2}} + \frac{\sqrt{x^2 - 8a^2}}{(\sqrt{x^2 - a^2})(\sqrt{x^2 - 8a^2})} - \frac{\sqrt{x^2 - 4a^2}}{(\sqrt{x^2 - a^2})(\sqrt{x^2 - 8a^2})}$$

$$\Rightarrow \frac{1}{\sqrt{4a^2}} + 0 = \frac{1}{\sqrt{4a^2}}$$

$$\lim_{x \rightarrow (-1)} \frac{1-k[x]}{x^2-1} = -\infty \quad \begin{matrix} x \rightarrow (-1)^+ \\ x \rightarrow (-1)^- \end{matrix} \quad \begin{matrix} \frac{1+k}{0^-} \\ \frac{1+2k}{0^+} \end{matrix} = -\infty \Rightarrow \begin{matrix} 1+k > 0 \Rightarrow k > -1 \\ 1+2k < 0 \Rightarrow k < -\frac{1}{2} \end{matrix} \quad (10)$$

$$\Rightarrow -1 < k < -\frac{1}{2} \Rightarrow -\pi < k\pi < -\frac{\pi}{2}, \quad -1 < \cos k\pi < 0$$

در ناحیه سوم