فاطمه حسینی

جزوه ریاضی ۱

تکلیف ۲۰

۱۵

۲۶ جمعه
اردیبهشت ۱۴۰۴
16 May 2025 | ۱۸ ذی القعده ۱۴۴۶

$\ominus \to -1 , 0 , -$

۱) $\lim_{x\to 2} \dfrac{\overbrace{3x-5}}{\underbrace{x^2+ax+b}} = -\infty \to a+b=?$ (۱

$(x-2)^2 = x^2-4x+4 \to \left.\begin{array}{l} a=-4 \\ b=4 \end{array}\right\} \boxed{a+b=0}$

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۲) $\lim_{x\to (\sqrt[3]{3})} \dfrac{x^{\sim \oplus}}{x^2+ax+b} = +\infty \sim \left[\dfrac{b}{a}\right]=?$ (۲

$0^+ \to (x-\sqrt[3]{3})^2 = (x^2 - 2\sqrt[3]{3}x + (\sqrt[3]{3})^2)$

$\begin{array}{l} a=-2\sqrt[3]{3} \\ b=(\sqrt[3]{3})^2 \end{array} \to \left[\dfrac{(\sqrt[3]{3})^2}{-2\sqrt[3]{3}}\right] = \left[\dfrac{\overset{+}{\sqrt[3]{3}}}{-2}\right] = \boxed{-1}$

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۳) $\lim_{x\to \left(\frac{1}{\sqrt{3}}\right)^{\oplus}} \dfrac{[-x]+a}{\left|\frac{\sqrt{3}}{2}x+a\right|+a} = -\infty \to [a]=?$ (۳

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$[-x]+a = -1+a \to \dfrac{-1+a}{\left|\frac{1}{2}+a\right|+a} = -\infty \to \left[-\frac{1}{6}\right] = \boxed{-1}$

$\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{3}} = \frac{1}{2}$

① $-\frac{1}{6}\ -\frac{1}{2}$

$\dfrac{-1+a}{\frac{1}{2}+2a} = \dfrac{-}{+} \checkmark = -\infty$

$2a = -\frac{1}{2} \to a = -\frac{1}{6}$

۴) $\lim_{x\to \left(-\frac{1}{2}\right)^{\oplus}} \dfrac{16x-\left[-\frac{2}{x^2}\right]}{24x+\left[\frac{3}{x^2}\right]} = ?$ (۴

$\frac{1}{x^2} > 4 \to \frac{-2}{x^2} < -8 \to \left[\frac{-2}{x^2}\right] = -9$

$\lim_{x\to(-\frac{1}{2})^+} \dfrac{16x+9}{24x+12} = \lim_{x\to(-\frac{1}{2})^+} \dfrac{-8+9}{0^+} = \dfrac{1}{0^+} = +\infty$

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$\dfrac{-8+8}{-12+12} = \dfrac{0}{0} \to \dfrac{16x+8}{24x+12} = \dfrac{8(\cancel{2x+1})}{12(\cancel{2x+1})} = \boxed{\dfrac{2}{3}}$

۵) $\lim\limits_{x \to 2^+} \dfrac{x^2-4}{x^3-[x^3]} = ?$ (۵

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$$\frac{x^2-4}{x^3-8} = \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+2x+4)} = \frac{4}{12} = \boxed{\frac{1}{3}}$$

$4+4+4$

۶) $\lim\limits_{x \to -8} \dfrac{x^2+10x+16}{12+6\sqrt[3]{x}} = ?$ ۵ (۶

$4+4-8=0$

ضرب لغیف

$$\frac{x^2+10x+16}{6(2+\sqrt[3]{x})} \times \frac{4-2\sqrt[3]{x}+\sqrt[3]{x^2}}{(4-2\sqrt[3]{x}+\sqrt[3]{x^2})} = \frac{(x+2)\cancel{(x+8)}(4-2\sqrt[3]{x}+\sqrt[3]{x^2})}{6\cancel{(8+x)}}$$

$$= \frac{-6\times 0}{6} = \boxed{0}$$

$\lim\limits_{x\to -8} \dfrac{x^2+10x+16}{12+6\sqrt[3]{x}} = \lim\limits_{x\to -8} \dfrac{(x+8)(x+2)}{6(2+\sqrt[3]{x})} \times \dfrac{\sqrt[3]{x^2}+4-2\sqrt[3]{x}}{\sqrt[3]{x^2}+4-2\sqrt[3]{x}} \rightarrow$

$\lim\limits_{x\to -8} \dfrac{(x+8)(x+2)\times 12}{6(x+8)} \rightarrow \lim\limits_{x\to -8} \dfrac{12(x+2)}{6} = 2\times(-6) = -12$

۷) $\lim\limits_{x\to 0^-} \dfrac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}} = ?$ (۷

$\lim\limits_{x\to 0^-} \dfrac{4x\times\sqrt{2}}{|\sin x|\times 2\sqrt{2}} = \lim\limits_{x\to 0^-} \dfrac{4x\times\sqrt{2}}{-2\sqrt{2}\sin x} \xrightarrow{\text{فشردگی}} \lim\limits_{x\to 0^-} \underbrace{\dfrac{x}{\sin x}} \times \dfrac{4\sqrt{2}}{-2\sqrt{2}} = -2$

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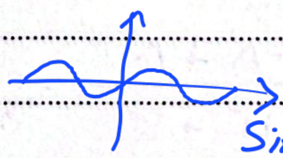
$$\frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{2+3x}+\sqrt{2-x}}{\sqrt{2+3x}+\sqrt{2-x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}$$

$$\frac{\cancel{2}+3x-\cancel{2}+x}{\underbrace{\sqrt{1-\cos^2 x}}(\underbrace{\sqrt{2+3x}+\sqrt{2-x}})} = \frac{\overset{0}{4x}(\sqrt{1+\cos x})}{(2\sqrt{2})\times(-\sin x)} = \boxed{0}$$

$\sqrt{2}+\sqrt{2} =$ $\quad 2\sqrt{2}\times -1 = -2\sqrt{2}$




$\sqrt{1-\cos^2 x} = |\sin x| =$

$-\sin x$

سوال ۸ حد مخرج صفر و حاصل حد عددی حقیقی و غیر صفر ← حد صورت نیز برابر صفر بوده

$$\lim_{x\to 0} k+\cos(\sqrt{a}x)=0 \to k+1=0 \to k=-1$$

$$\lim_{x\to 0}\frac{-1+\cos(\sqrt{a}x)}{-x^2} \xrightarrow{\text{هم ارزی}} \lim_{x\to 0}\frac{-1+1-\frac{(\sqrt{a}x)^2}{2}}{-x^2}=\lim_{x\to 0}\frac{\frac{-ax^2}{2}}{-x^2}=\frac{a}{2}\to\frac{a}{2}=3\to a=6$$

$$\frac{a}{k}=\frac{6}{-1}=-6$$

سوال ۹

$$\lim_{x\to 3a^+}\frac{2\sqrt{x-3a}}{\sqrt{x-3a}\sqrt{x+3a}}+\frac{3\sqrt{x}-3\sqrt{3a}}{\sqrt{x-3a}\sqrt{x+3a}}=\lim_{x\to 3a^+}\left(\frac{2}{\sqrt{6a}}+\frac{3(\sqrt{x}-\sqrt{3a})}{\sqrt{x-3a}\sqrt{x+3a}}\times\frac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}+\sqrt{3a}}\right)\to$$

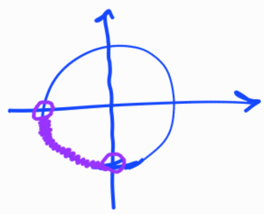
$$\lim_{x\to 3a^+}\left(\frac{2}{\sqrt{6a}}+\frac{2(x-3a)}{(\sqrt{x-3a}\sqrt{x+3a})(\sqrt{x}+\sqrt{3a})}\right)=\lim_{x\to 3a^+}\left(\frac{2}{\sqrt{6a}}+\frac{3\sqrt{x-3a}\to 0}{(\sqrt{x}+\sqrt{3a})(\sqrt{x+3a})}\right)=\lim_{x\to 3a^+}\frac{2}{\sqrt{6a}}+0$$

$$\to\frac{\sqrt{4}}{\sqrt{6a}}=\sqrt{\frac{4}{6a}}=\sqrt{\frac{2}{3a}}$$

سوال ۱۰

$$\begin{cases}\lim_{x\to(-1)^+}\frac{1-k[x]}{x^2-1}=\lim_{x\to(-1)^+}\frac{1-k(-1)}{0^-}=\frac{1+k}{0^-}=-\infty\to 1+k>0\to k>-1 & (\mathrm{I})\\[2ex] \lim_{x\to(-1)^-}\frac{1-k[x]}{x^2-1}=\lim_{x\to(-1)^-}\frac{1-k(-2)}{0^+}=\frac{1+2k}{0^+}=-\infty\to 1+2k<0\to k<-\frac{1}{2} & (\mathrm{II})\end{cases}$$

$$(\mathrm{I})\cap(\mathrm{II})\to -1<k<-\frac{1}{2}\to\begin{cases}-\pi<k\pi<-\frac{\pi}{2}\\ -1<\cos k\pi<0\end{cases}$$



ناحیه سوم