۱-

$2x-5 \xrightarrow{x\to 2} 2\times 2-5=-1$ پس $\Rightarrow x^2+ax+b = 0^+$

$(x-2)^2 = x^2+ax+b \Rightarrow b=4 \quad a=-4 \quad a+b=0$

۲-

$x=\sqrt[3]{4} \Rightarrow x^2+ax+b=0^+ \quad (x-\sqrt[3]{4})^2 = x^2+ax+b$

$\Rightarrow a=-2\sqrt[3]{4} \quad b=\sqrt[3]{16} \quad [\frac{b}{a}] = \frac{\sqrt[3]{16}}{\sqrt[3]{-32}} = -1$

۳-

$x\to \frac{1}{\sqrt{3}}^+ \Rightarrow \frac{\sqrt{3}}{3}\times x = \frac{\sqrt{3}}{3}\times\frac{1}{\sqrt{3}} = \frac{1}{3}^+ \qquad \frac{a-1}{|\frac{1}{3}+a|+a} = -\infty$

$[-x] = [-\frac{1}{\sqrt{3}}^-] = -1$

① $a<-\frac{1}{3} \Rightarrow \frac{a-1}{-\frac{1}{3}-a+a} = \frac{a-1}{-\frac{1}{3}} = +\infty$ a نیز منفی است پس حد مثبت می‌شود

② $a>-\frac{1}{3} \quad \frac{a-1}{\frac{1}{3}+2a} = -\infty \Rightarrow \frac{1}{3}+2a=0^+ \Rightarrow a=(-\frac{1}{6})^+ \Rightarrow [a]=[-\frac{1}{6}^+]=-1$

۴-

$x\to -\frac{1}{2} \Rightarrow x^2<\frac{1}{4} \Rightarrow \frac{1}{x^2}>4 \Rightarrow \frac{-2}{x^2}<-8 \Rightarrow [\frac{-2}{x^2}]=-9$

$\frac{3}{x^2}>12 \Rightarrow [\frac{3}{x^2}]=12$

$\lim \frac{16x-[\frac{-2}{x^2}]}{24x+[\frac{3}{x^2}]} = \frac{-8+9}{-12^+ +12} = \frac{1}{0^+} = \infty$

۵

$\lim_{x\to 2^+} \frac{x^2-4}{x^3-\underbrace{\cancel{[x^3]}}_{8}} = \frac{2x}{3x^2} = \frac{4}{12} = \frac{1}{3}$

۶

$\lim_{x\to -8} \frac{x^2+10x+16}{12+6\sqrt[3]{x}} = \frac{2x+10}{6\cdot\frac{1}{3\sqrt[3]{x^2}}} = \frac{-6}{6\times\frac{1}{12}} = \frac{-6}{\frac{1}{2}} = -12$

۷-

$\lim_{x\to 0^-} \frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} \times \frac{\sqrt{2+3x}+\sqrt{2-x}}{\sqrt{2+3x}+\sqrt{2-x}}$

$= \frac{\overbrace{(2+3x-(2-x))}^{4x}(\sqrt{1+\cos x})}{\underbrace{(\sqrt{1-\cos^2 x})}_{-x}(\sqrt{2+3x}+\sqrt{2-x})} = \frac{-4\sqrt{2}}{2\sqrt{2}} = -2$

Arman

$=\sqrt{\sin^2 x} = |\sin x| = -\sin x = -x$

8

$$\cos x \underset{x\to 0}{\sim} 1-\frac{x^2}{2} \qquad \cos\sqrt{a}x \sim 1-\frac{ax^2}{2}$$

$$\lim_{x\to 0}\frac{k+\cos(\sqrt{a}\,x)}{kx^2} = \frac{k+1-\frac{ax^2}{2}}{kx^2} = 3 \Rightarrow k+1=0 \Rightarrow k=-1$$

$$\Rightarrow \frac{\frac{-ax^2}{2}}{-x^2} = \frac{a}{2} = 3 \Rightarrow a=6 \qquad \frac{a}{k} = -6$$

9

$$\lim_{x\to 3a^+}\frac{2\sqrt{x-3a}+3\sqrt{x}-\sqrt{27a}}{\sqrt{x^2-9a^2}} = \lim_{x\to 3a^+}\frac{2\sqrt{x-3a}}{\sqrt{x^2-9a^2}} + \frac{3\sqrt{x}-3\sqrt{3a}}{\sqrt{x^2-9a^2}} = \frac{0}{0}$$

$$= \frac{2\sqrt{x-3a}}{\sqrt{x-3a}\sqrt{x+3a}} + \frac{3\sqrt{x}-3\sqrt{3a}}{\sqrt{x-3a}\sqrt{x+3a}} \times \frac{\sqrt{x}+\sqrt{3a}}{\sqrt{x}+\sqrt{3a}} = \frac{2}{\sqrt{x+3a}} + \frac{3(x-3a)}{\sqrt{x-3a}\sqrt{x+3a}(\sqrt{x}+\sqrt{3a})}$$

$$= \frac{2}{\sqrt{x+3a}} + \frac{3\sqrt{x-3a}}{\sqrt{x+3a}(\sqrt{x}+\sqrt{3a})} = \frac{2}{\sqrt{6a}} + 0 = \frac{2}{\sqrt{6a}}$$

10

$$x\to -1^+ \nearrow \lim_{x\to -1^+}\frac{1+k}{x^2-1} = \frac{1+k}{0^-} = -\infty \Rightarrow 1+k>0 \Rightarrow k>-1$$

$$\lim_{x\to -1}\frac{1-k[x]}{x^2-1} = -\infty$$

$$x\to -1^- \searrow \lim_{x\to -1^-}\frac{1+2k}{x^2-1} = \frac{1+2k}{0^+} = -\infty \Rightarrow 1+2k<0 \Rightarrow 2k<-1$$

$$k<-\frac{1}{2}$$

$$\Rightarrow -1<k<-\frac{1}{2} \Rightarrow -\pi<k\pi<-\frac{\pi}{2}$$

ناحیه سوم