

$$\begin{aligned} & \Rightarrow (2a^2 + (m-2)a + a^2) = 0 \Rightarrow a^2(2a + m - 2) = 0 \\ & \quad \begin{cases} a = 0 \\ 2a + m - 2 = 0 \Rightarrow m = 2 - 2a \end{cases} \\ & \Rightarrow \sqrt{4x^2 + (m+1)x + \frac{m}{4}} = 0 \\ & \Rightarrow 4a^2 + (2-2a)a + \frac{2-2a}{4} = 5a^2 + 2a + 1 = 0 \\ & \quad (2a+1)^2 = 0 \Rightarrow a = -\frac{1}{2}, m = 2 \\ & \Rightarrow \frac{\sqrt{\frac{1}{4}}}{|2x^2 - \frac{x}{2} + \frac{1}{2}|} = \frac{2 \tan b}{\sqrt{4x^2 + 4x + 1}} \Rightarrow \frac{1}{\sqrt{4x^2 + 4x + 1}} = \frac{2 \tan b}{\sqrt{4x^2 + 4x + 1}} \\ & \quad \Rightarrow \frac{1}{\sqrt{4x^2 + 4x + 1}} = \frac{2 \tan b}{\sqrt{4x^2 + 4x + 1}} \Rightarrow \tan b = \frac{1}{2} \Rightarrow b = \frac{\pi}{4} \end{aligned}$$

$$\begin{aligned} & f(x) = \frac{x^2 + ax + b}{x-1} \xrightarrow{x=1} 1 + a + b = 0 \Rightarrow a + b = -1 \\ & ax^2 - ax + b = 0 \Rightarrow a - a + b = 0 \Rightarrow a - b = a \\ & \quad \begin{cases} a = 2 \\ b = -3 \end{cases} \\ & \left(\frac{b-a}{2} \right) = \left(\frac{-3-2}{2} \right) = \left(-\frac{5}{2} \right) = -2.5 \end{aligned}$$

$$\begin{aligned} & f(x) = \begin{cases} \tan\left(\frac{(x+1)\pi}{4}\right) & x \leq 1 \\ \frac{b(x-1)}{a(1-x)} & 1 < x < a \end{cases} \\ & \Rightarrow \frac{b(x-1)}{a(1-x)} = \frac{1}{1-x} \Rightarrow \frac{b(x-1)}{a(1-x)} = \frac{1}{1-x} \Rightarrow \frac{b}{a} = -1 \\ & \Rightarrow ab = 1 \left(-\frac{1}{a} \right) = -1 \Rightarrow a = 1, b = -1 \end{aligned}$$

$$\begin{aligned} & f(x) = a[x+1] + b[n+[a+1]] \Rightarrow f(x) = a[x] + a + b[x] + b[a] + b \\ & f(a) = a[a] + a + b[a] + b[a] + b \\ & \Rightarrow a[a] + a - a[a] - a[a] - a = -a[a] \\ & \Rightarrow \frac{a[a]}{f(a)} = \frac{a[a]}{-a[a]} = -1 \end{aligned}$$

$$\begin{aligned} & f(x) = b[x^2 - ax] - xa \xrightarrow{x=0} 0 = b[0] - 0 \Rightarrow b = 0 \Rightarrow f(x) = -xa \\ & \frac{a}{f(b)} = \frac{a}{-ba} = \left(-\frac{1}{b} \right) \end{aligned}$$

$$f(n) = \begin{cases} \frac{n^r + mn + n}{a - n} & n \neq a \\ r & n = a \end{cases} \xrightarrow{\text{محدود}} (n-a) \text{ بر } r \text{ بخش پذیر!}$$

$$\Rightarrow \frac{(n-a)(n-r)}{-(n-a)} = -(n-r) \xrightarrow{n=a} -(a-r) = r \Rightarrow a = r$$

$$(n-a)(n-r) = n^r - ra n + a^r = n^r + mn + n \begin{cases} m = -ra \Rightarrow m = -r \\ n = ra^r \Rightarrow n = 1 \end{cases} \rightarrow n-m = 1 - (-r) = \boxed{r}$$

$$f(n) = \begin{cases} (1-a)[n] + (ra^r - 1)[-n] & n \notin \mathbb{Z} \\ b \sin\left(\frac{\pi}{a}\right) & n \in \mathbb{Z} \end{cases}$$

$$\rightarrow \frac{a}{b} = \frac{\frac{r}{\frac{1}{r}}}{\frac{1}{r}} = \boxed{r}$$

$$n = 0^+ \Rightarrow (1-a)(0) + (ra^r - 1)(-1) = 1 - ra^r$$

$$n = 0 \Rightarrow b \sin\left(\frac{\pi}{a}\right)$$

$$n = 0^- \Rightarrow (1-a)(-1) + (ra^r - 1)(0) = a - 1$$

$$\begin{aligned} 1 - ra^r &= a - 1 \Rightarrow ra^r + a - r = 0 \\ \text{فرض } a &= -1 \\ 1 - ra^r &= 1 - r = -r \\ a - 1 &= -1 - 1 = -2 \\ a - 1 &= -1 - 1 = -2 \\ b \sin\left(\frac{\pi}{a}\right) &= b \sin(\pi) = 0 \\ b \sin\left(\frac{\pi}{a}\right) &= b \sin\left(\frac{\pi}{r}\right) = -b \end{aligned}$$

$$f(n) = \begin{cases} \frac{n^r + mn - m - 1}{m(n^r - 1) + |n - 1|} & n \neq 1 \\ \frac{m}{m+r} & n = 1 \end{cases} \xrightarrow{n \neq 1} \frac{n^r + mn - 1 - m}{(m+1)n - (1+m)}$$

$$\Rightarrow \frac{(n-1)(n+m+1)}{(n-1)(m+1)+|n-1|} \xrightarrow{n=1} \frac{1+m+1}{m+1} = \frac{m}{m+r}$$

$$m < -r \Rightarrow -m^r < m - r = rm^r + m \Rightarrow rm^r + m + r = 0 \Rightarrow \Delta < 0 \times$$

$$m > -r \Rightarrow m^r < m + r = rm^r + m \Rightarrow m^r - rm - r = 0 \Rightarrow (m-r)(m+1) = 0$$

$$\begin{aligned} m &= r \Rightarrow \lim_{n \rightarrow \frac{1}{r}} f(n) = \frac{1 + \frac{1}{r} + 1}{\left(\frac{1}{r} + \frac{1}{r} + 1\right)} = \frac{\frac{r}{r}}{\frac{r}{r}} = \boxed{\frac{r}{r}} \\ m &= -1 \Rightarrow \lim_{n \rightarrow -1} f(n) = \frac{-1 - 1 + 1}{(-1 - 1 + 1)} = \boxed{1} \end{aligned}$$

$$f(n) = \frac{\sqrt{r} |n+1|}{n^r + (m-r)n^r + a^r} \xrightarrow{n=1} \frac{\sqrt{r} |1+1|}{1 + (m-r)1 + a^r}$$

$$\mathbb{R} - \{a\} \xrightarrow{\text{محدود}} \frac{1}{a} = a \Rightarrow \sqrt{r} |a+1| = 0$$

$$\lim_{n \rightarrow a} f(n) = \frac{\sqrt{r} |a+1|}{a^r + (m-r)a^r + a^r} = \frac{\sqrt{r} |a+1|}{a^r + 1}$$

$$\begin{aligned} a(a+1) &= 0 \Rightarrow a=0 \Rightarrow \lim_{n \rightarrow 0} f(n) = 0 \\ a &= -1 \checkmark \Rightarrow \lim_{n \rightarrow -1} f(n) = \frac{\sqrt{r} |-1+1|}{1-1+1} = 0 \\ m-r &= 0 \\ m &= r \end{aligned}$$

$$\frac{\sqrt{r} |a+1|}{a^r + (m-r)a^r + a^r} = \frac{\sqrt{r}}{1+1+1} = \boxed{\frac{\sqrt{r}}{3}}$$

$$n=0 \rightarrow \frac{1}{a} = \frac{ra-1}{ra+r}$$

$$f(n) = \begin{cases} \frac{n^r + n}{n^r + a|n|} = \frac{|n|(|n+1|)}{|n|(1+a)} = \frac{|n+1|}{1+a} & n \neq 0 \\ \frac{ra-1}{ra+r} & n = 0 \end{cases}$$

$$\lim_{n \rightarrow \frac{1}{r}} f(n) = \frac{\frac{1}{r} + 1}{\frac{1}{r} + r} = \boxed{\frac{r}{a}}$$

$$\lim_{n \rightarrow -r} f(n) = \frac{r+1}{r-\frac{1}{r}} = \boxed{\frac{r}{r}}$$

$$\begin{aligned} ra^r - a &= ra+r \Rightarrow ra^r - ra - r = 0 \\ a &= \frac{r \pm \sqrt{r^2 + 4r}}{2} \end{aligned}$$

$$m=r \rightarrow \frac{|a^r + a - a|}{r|a-1||a+1| + |a-1|} = \frac{|a-1|(|a+1|)}{|a-1|(r|a+1| + 1)} \rightarrow \lim_{a \rightarrow \frac{1}{r}} \frac{\frac{1}{r} + 1}{\frac{1}{r} + r} = \frac{\frac{r}{r}}{\frac{r}{r}} = \boxed{\frac{r}{r}}$$