آیلا مهردنیا

۱.

$|2a^3 + (m-3)a^2 + a^2| = 0 \to a^2(2a + (m-2)) = 0$

$a = 0$ غ ق ق $\qquad 2a = -m + 2 \to m = 2 - 2a$

$\sqrt{4x^2 + (m+3)x + \frac{m}{2}} = 0 \to 4a^2 + (m+3)a + \frac{m}{2} = 0 \to 4a^2 + (5-2a)a + \frac{2-2a}{2}$

$= 4a^2 + 4a + 1 = 0 \to (2a+1)^2 = 0 \to a = \frac{-1}{2} \to m = 3$

$\to \frac{\sqrt{\frac{3}{2}}\,|2x+1|}{(2x+\frac{1}{2})|x^2 - \frac{x}{2} + \frac{1}{4}|} \to \frac{\sqrt{\frac{3}{2}}}{|x^2 - \frac{x}{2} + \frac{1}{4}|} = \frac{2\tan b}{\sqrt{-x}} \quad x > -\frac{1}{2} \to \frac{\sqrt{\frac{3}{2}}}{\frac{3}{4}} = \frac{2\tan b}{\sqrt{\frac{1}{2}}}$

$\tan b = \frac{\sqrt{3}}{3} \to b = \frac{\pi}{4}$

۲.

$\frac{x^2 + ax + b}{x-1} \xrightarrow{x=1} 1 + a + b = 0 \to a + b = -1$

$5x^2 - ax + b = 0 \xrightarrow{x=1} 5 - a + b = 0 \to -a + b = -5$

$2b = -6 \to b = -3 \,/\, a = 2$

$\left[\frac{b-2a}{3}\right] = \left[\frac{-3-4}{3}\right] = \left[\frac{-7}{3}\right] = -3$

۳.

$$f(x) = \begin{cases} \tan\frac{(2x+1)\pi}{4} & 1 \le x \\ \frac{|x^2+x-2|}{a(1-x)} & 1 < x < 5 \\ b(x - [-x]) & x \ge 5 \end{cases} \to \frac{|(x+2)(x-1)|}{a(1-x)} \to \frac{-(x+2)}{a}$$

$-2 \quad 1$

$+ \quad - \quad +$

$x = 1 \to \tan\frac{3\pi}{4} = \frac{-3}{a} \to -1 = \frac{-3}{a} \to a = 3$

$x = 5 \to \frac{-7}{-3} = b(5 - (-5)) \to \frac{-7}{3} = 10b \to b = \frac{-7}{30}$

آیلا فردوئنیا

۴.

$f(x) = a[x] + a + b[x] + b[a] + b$

$x \to a^+ : a + b[a] + b \longrightarrow a + b[a] + b = b[a]$

$x \to a^- : \cancel{-a + a} - b + b[a] + b = b[a] \longrightarrow a + b = 0 \longrightarrow a = -b \,/\, b = -a$

$f(a) = a[a] + \cancel{a} - a[a] - a[a] \cancel{- a} = -a[a]$

$\dfrac{a[a]}{-a[a]} = -1$

۵.

$f(x) = b[x^2 - ax] - 2a$ — ضریب جزء صحیح — $b = 0$

$f(x) = -2a \longrightarrow \dfrac{a}{-2a} = -\dfrac{1}{2}$

۶.

$$f(x) = \begin{cases} \dfrac{x^2 + mx + n}{a - x} & x \neq a \\ 2 & x = a \end{cases}$$

$\dfrac{(x-a)(x-2a)}{-(x-a)} = 2a - x \longrightarrow -(a - 2a) = 2 \longrightarrow a = 2$

$(x-a)(x-2a) = x^2 - 3ax + 2a^2 = x^2 + mx + n$

$\longrightarrow m = -3a \longrightarrow m = -6$

$\longrightarrow n = 2a^2 \longrightarrow n = 8$

$\longrightarrow n - m = 8 - (-6) = 14$

۷.

$0^+ : 1 - 3a^2$

$0 : b \sin\left(\dfrac{\pi}{a}\right)$

$0^- : a - 1$

$1 - 3a^2 = a - 1 \longrightarrow 3a^2 + a - 2 = 0 \longrightarrow -1$ غ ق ق ، $\dfrac{2}{3}$ ✓

$-\dfrac{1}{3} = b \sin\left(\dfrac{3\pi}{2}\right) \longrightarrow -\dfrac{1}{3} = -b \longrightarrow -3b = -1$

$b = \dfrac{1}{3}$

$\dfrac{a}{b} = \dfrac{\frac{2}{3}}{\frac{1}{3}} = 2$

آیکلا مفردشنیا

۸.

$$\frac{|x-1|(x+m+1)}{|x-1|(m|x+1|+1)} = \frac{x+m+1}{(m|x+1|+1)} \xrightarrow{x=1} \frac{m+2}{2m+1}$$

$$\frac{m+2}{2m-1} = \frac{m}{m+2} \to m<-2 \;,\; -m^2-4m-4 = 2m^2+m \to 3m^2+5m+4=0 \to \Delta<0 \;\text{غ ق ق}$$

$$m>-2 \;,\; m^2+4m+4 = 2m^2+m \to m^2-3m-4=0$$

$$m=-1 \to \lim_{x\to -1} f(x) = \frac{|-1-1+1|}{(-1\times 0+1)} = 1$$

$$m=4 \quad \lim_{x\to \frac{1}{4}} f(x) = \frac{|\frac{1}{4}+\frac{1}{4}+1|}{|4|\frac{1}{4}\times\frac{5}{4}+1|} = \frac{24}{19}$$

۹.

$$f(x) = \frac{\sqrt[3]{|ax+a|}}{|x^3+(m-2)x^2+a^2|} \sim |-x-1| = |x+1|$$

$$\mathbb{R}-\{a\} \longrightarrow a \longrightarrow \sqrt[3]{|a^2+a|} = 0$$

$\lim_{x\to a} f(x)$ ریشه مشترک

$$a(a+1)=0 \sim a=0 \;\text{غ ق ق}$$

$$a=-1 \underset{x=-1}{\sim} |-1+(m-2)|+1| = 0 \to m=2$$

$$f(x) = \frac{\sqrt[3]{|-x-1|}}{|x^3+(2-2)x^2+1|} = \frac{\sqrt[3]{|x+1|}}{|x^3+1|} \to \lim_{x\to -1} f(x) = \frac{\sqrt[3]{|1-1|}}{|-1+1|} = \frac{0}{0}$$

$$\longrightarrow \frac{|x+1|}{|x+1||x^2-x+1|} = \frac{1}{|1+1+1|} = \frac{1}{3}$$

-۱۰

$$x=0 \longrightarrow \frac{1}{a} = \frac{2a-1}{2a+2} \longrightarrow 2a^2-a = 2a+2 \longrightarrow 2a^2-3a-2=0$$

$$a = \frac{3 \pm \sqrt{9+16}}{4} \to 2 \;,\; -\frac{1}{2}$$

$$\lim_{x\to \frac{1}{2}} f(x) \longrightarrow \frac{\frac{1}{4}+\frac{1}{2}}{\frac{1}{4}+1} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}$$

$$\lim_{x\to -2} f(x) \longrightarrow \frac{4+2}{4-1} = \frac{6}{3} = 2$$