فاطمه سلطان اکبری ۴

$$f(x) = \begin{cases} \dfrac{\sqrt{6x^2+(m+3)x+\frac{m}{2}}}{|2x^3+(m-3)x^2+a^2|} & x \neq a \\[2ex] \dfrac{2\tan b}{\sqrt{-x}} \end{cases}$$

$$\lim_{x\to a} \frac{\sqrt{6x^2+6x+\frac{3}{2}}}{|2x^3+a^2|} = \frac{\sqrt{6(x^2+x+\frac{1}{4})}}{|2x^3+a^2|} = \frac{\sqrt{6}\,|x+\frac{1}{2}|}{|2x^3+a^2|} \quad (1)$$

$a \leftarrow$ تنها ریشه زیر رادیکال و باید مخرج را هم صفر کند

$$\Delta = 0 \rightarrow (m+3)^2 - 4\left(\frac{m}{2}\right)\times 6 = 0 \rightarrow m^2+6m+9-12m=0 \rightarrow m^2-6m+9=0 \rightarrow m=3$$

$$f(x) = \begin{cases} mx^2+3x+2 & x<a \Rightarrow \lim\limits_{x\to a^-} f(x) = am^2+3a+2 \\ \tan x & x=a \rightarrow f(a) = \tan a \\ bx-2 & x>a \Rightarrow \lim\limits_{x\to a^+} f(x) = ba-2 \end{cases}$$

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$$2a^3+a^2=0 \rightarrow a^2(2a+1)=0 \rightarrow \begin{cases} a=0 \rightarrow \frac{\text{عدد}}{0} \ ✗ \\ a=-\frac{1}{2} \rightarrow \dfrac{\sqrt{6}\,|x+\frac{1}{2}|}{|2x^3+\frac{1}{4}|} = \frac{0}{0} \ ✓ \end{cases}$$

$$\Rightarrow f(a) = \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$$

$$\lim_{x\to -\frac{1}{2}} \frac{\sqrt{6}\,|x+\frac{1}{2}|}{2|x^3+\frac{1}{8}|} = \frac{\sqrt{6}\,|x+\frac{1}{2}|}{2|x+\frac{1}{2}|\,|x^2-\frac{1}{2}x+\frac{1}{4}|} = \frac{2\sqrt{6}}{3}$$

$$\Rightarrow \tan a = am^2+3a+2$$

$$\tan a = ba-2 \qquad \tan\frac{\pi}{4} = 1 \Rightarrow \frac{\pi}{4} = a$$

$$\Rightarrow \tan a = ba-2$$

$$\Rightarrow 1 = b\left(\frac{\pi}{4}\right) - 2 \Rightarrow 3 = \left(\frac{\pi}{4}\right)b \Rightarrow b = \frac{12}{\pi}$$

$$\frac{2\tan b}{\sqrt{-(-\frac{1}{2})}} = \frac{2\sqrt{6}}{3} \rightarrow \tan b = \frac{\sqrt{3}}{3} \rightarrow \boxed{b = \frac{\pi}{6}}$$

$$\frac{k\pi}{a} = x \Rightarrow \lim_{x\to \frac{k\pi}{a}} f(x) = f\left(\frac{k\pi}{a}\right) \quad (2)$$

$$f(x) = \sin(ax) - a + x \Rightarrow \lim_{x\to \frac{k\pi}{a}} f(x) = \overbrace{\sin(k\pi)}^{0} - a + \frac{k\pi}{a} \Rightarrow = -a + \frac{k\pi}{a}$$

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$$-a + \frac{k\pi}{a} = b\left(\frac{k\pi}{a}\right) \Rightarrow \frac{\pi}{a} = \frac{b\pi}{a} \Rightarrow b = 1$$

$$a = 1 \Rightarrow \frac{b}{a} = 1$$

Date: Sub: $\begin{cases}1+a+b=0\\ 8-a+b=0\end{cases} \to \begin{matrix} b=-3 \\ a=2\end{matrix} \to \left[\frac{-3-2(2)}{3}\right] = \left[\frac{-7}{3}\right] = -3$

(2) $2ax^2 - b + 2 = 0$, $\frac{x^2 + ax + b}{x} = f(x)$

$x=2 \Rightarrow 2a(2)^2 - b + 2 = 0$ $\qquad b=0$

$\Rightarrow 8a - b + 2 = 0 \Rightarrow 8a - b = 3$ $\qquad$ تعریف $x=0$ نداریم!

(1) $\Rightarrow b=0 \Rightarrow 8a = 3 \Rightarrow a = \frac{3}{8}$ $\qquad \Rightarrow [b-a] = 0$

$\frac{3}{8} \cdot 0 = \frac{3}{8}$

(3) $x \le 1 \overset{f(1)=}{\Rightarrow} \tan 1 \Rightarrow \lim_{x\to 1} (ax^2 - x + b) = 1(a^2) - 1 + b = a - b$

$a + b - 1 = \tan 1$ (1)

$x \ge 2 \Rightarrow f(2) = 4 - b = -1 \Rightarrow \lim_{x\to 2} (ax^2 - x + b) = 4a - 2 + b$

$a = 3$ $\qquad b = -\frac{7}{3}$ $\qquad \Rightarrow 4a - 2 + b = -1$

$ab = -\frac{7}{10}$ $\qquad \Rightarrow 4a + b = 1$

$\tan 1 + 1 = b + a$

$4a + b = 1 \Rightarrow -\tan 1 = 3a \Rightarrow a = \frac{-\tan 1}{3}$

$\Rightarrow \left(\frac{-\tan 1}{3}\right)4 + b = 1 \Rightarrow b = \frac{\tan 4}{3} + 1 = b$

$\Rightarrow b = 1$

(4) $\Delta < 0 \Rightarrow b^2 - 4a < 0$

$\Rightarrow f(a) = a(a)^2 + b(a) + a + 1 \Rightarrow f(a) = a^3 + ab + 1$ (9)

$\Rightarrow f(a) = a^3 + a + 1 \Rightarrow a^3 + a + 1$

(5) $f(b) = b(b^2) - (b)a - a \Rightarrow f(b) = b^3 - ab - a \Rightarrow b^3 - ab - a$ (a) ?

$b=0 \to f(a) = -2a \to \frac{a}{f(b)} = \frac{a}{-2a} = \frac{-1}{2}$

(6) $\lim_{x\to a} f(x) = f(a) \Rightarrow x \ne a \Rightarrow f(x) = mx^2 + nx$ (-8)

$\lim_{x\to a} f(x) = ma^2 + na$

$a = 0 \Rightarrow n(0) + m^2(0) = 2$

$\Rightarrow 2 \ne 0$

$\lim_{x\to a} (ma^2 + 2a) = ma^2 + 2a \Rightarrow na^2 + 2a = 2$ $\qquad n - m = 2 - \frac{2a - 2}{a^2}$

$a = 0 \Rightarrow 0 + 0 \ne 2$ ✗

Elipon $\qquad \Rightarrow ma^2 + 2a = 2 \Rightarrow m = \frac{2 - 2a}{a^2}$

Date: Sub:

$\lim_{x\to 1} f(x) = f(1) \Rightarrow \lim_{x\to 1} (mx^3 - mx - 1) =$ ۱

$m^3(1) - m(1) = 1$

$\Rightarrow 1 + m = f(1)$ ۰

$\Rightarrow m+1 = -1 \Rightarrow \cancel{m = -2} \Rightarrow \lim_{x\to 1} f(x) = -1 = -1$

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$x^2 - 3x + 2 = (x-1)(x-2) \Rightarrow a = \cancel{1, 2}$ ۰

$\Rightarrow a = 1 \Rightarrow \lim_{x\to 1} f(x) = \frac{1}{x-2} = -1$

$\Rightarrow a = 2 \Rightarrow \lim_{x\to 2} = \pm\infty \Rightarrow$ ۲

۱۰

$\lim_{x\to a} f(x) = f(a) \Rightarrow \lim_{x\to a} (x^2 + a) = a^2 + a$ ۰

$\Rightarrow f(a) = 2a + 1$

$\Rightarrow a^2 + a = 2a + 1 \Rightarrow a^2 - a - 1 = 0$

$\Rightarrow a = \frac{1 \pm \sqrt{5}}{2}$

$\Rightarrow 2a + 1$

سوال ۴

$f(x) = a[x] + a + b[x] + b[a] + b$

$x \to 0^+ : a[0^+] + a + b[0^+] + b[a] + b = a + b[a] + b$

$x \to 0^- : a[0^-] + a + b[0^-] + b[a] + b = b[a]$

چون در R پیوسته $\rightarrow a + b[a] + b = b[a] \rightarrow a = -b$

$\rightarrow \frac{a[a]}{f(a)} = \frac{a[a]}{b[a]} = \frac{a}{b} = \frac{-b}{b} = -1$

سوال ۶

$f(2a)=0 \rightarrow x=2a \rightarrow$ ریشه صورت

$x=a \rightarrow$ ریشه صورت + ریشه مخرج

$$f(x)=\frac{(x-a)(x-2a)}{-(x-a)}=-(x-2a) \rightarrow f(x)=\begin{cases}-(x-2a) & x\neq a\\ 2 & x=a\end{cases}$$

$$f(a)=-(a-2a)=2 \rightarrow a=2$$

$$f(x)=\frac{(x-a)(x-2a)}{-(x-a)}=\frac{(x-2)(x-4)}{-(x-2)}=\frac{x^2-6x+8}{-(x-2)}=\frac{x^2+mx+n}{-(x-2)}$$

$$m=-6\ ,\ n=8 \rightarrow n-m=8-(-6)=14$$

سوال ۷

$$\text{حد راست}=(1-a)[Z^+]+(3a^2-1)[-Z^+]=(1-a)Z+(3a^2-1)(-Z-1)\quad (I)$$

$$\text{حد چپ}=(1-a)[Z^-]+(3a^2-1)[-Z^-]=(1-a)(Z-1)+(3a^2-1)(-Z)\quad (II)$$

$$I,II \rightarrow \lim_{x\to Z^+} f(x)=\lim_{x\to Z^-} f(x) \longrightarrow -1+a=-3a^2+1 \rightarrow 3a^2+a-2=0 \rightarrow \begin{cases}a=-1 & \times\\ a=\frac{2}{3}\end{cases}$$

$$a=\frac{2}{3} \rightarrow b\sin\left(\frac{\pi}{\frac{2}{3}}\right)=-b \rightarrow -b=-\frac{1}{3} \rightarrow b=\frac{1}{3} \qquad \frac{a}{b}=\frac{\frac{2}{3}}{\frac{1}{3}}=2$$

سوال ۸

$$\lim_{x\to\frac{1}{m}} \frac{|(x-1)(x+m+1)|}{|(x-1)(x+1)|+|x-1|}=\frac{|x-1||(x+m+1)|}{|x-1|(m|x+1|+1)}=\frac{|m+2|}{2m+1}=\frac{m}{m+2}$$

$$m\geq -2 \rightarrow m^2+4m+4=2m^2+m \rightarrow m^2-3m-4=0 \rightarrow \begin{cases}m=-1\\ m=4\end{cases}$$

$$m=4 \rightarrow \frac{|x^2+4x-5|}{4|x-1||x+1|+|x-1|}=\frac{|x-1||x+5|}{|x-1|(4|x+1|+1)} \rightarrow \lim_{x\to\frac{1}{4}} \frac{5+\frac{1}{4}}{4\times\frac{5}{4}+1}=\frac{\frac{21}{4}}{6}=\frac{7}{8}$$

$$m=-1 \rightarrow \frac{|x^2-x|}{-|x-1||x+1|+|x-1|}=\frac{|x||x-1|}{|x-1|(-|x+1|+1)} \quad \times$$

سوال ۹

$x=a \rightarrow$ ریشه صورت و مخرج

$$a^2+a=0 \rightarrow a(a+1)=0 \rightarrow \begin{cases}a=0 & \times\\ a=-1 & \checkmark\end{cases}$$

$$a^3+a(m-2)+a^2=0 \xrightarrow{a=-1} -1-m+2+1=0 \rightarrow m=2$$

$$\lim_{x\to -1} \frac{\sqrt{3}\,|-x-1|}{|x^3+1|}=\frac{\sqrt{3}\,|-(x+1)|}{|(x+1)(x^2-x+1)|}=\frac{\sqrt{3}}{3}$$

سوال ۱۰

$$\lim_{x\to 0^+}\frac{x^2+|x|}{x^2+a|x|}=\frac{0}{0}\rightarrow \lim_{x\to 0^+}\frac{x^2+x}{x^2+ax}=\lim_{x\to 0^+}\frac{x(x+1)}{x(a+x)}=\frac{1}{a}$$

$$\lim_{x\to 0^-}\frac{x^2+|x|}{x^2+a|x|}=\frac{0}{0}\rightarrow \lim_{x\to 0^-}\frac{x^2-x}{x^2-ax}=\lim_{x\to 0^-}\frac{x(x-1)}{x(x-a)}=\frac{-1}{-a}=\frac{1}{a}$$

$$f(0)=\frac{2a-1}{2a+2}=\frac{1}{a}\rightarrow 2a^2-a=2a+2\rightarrow 2a^2-3a-2=0\rightarrow \begin{cases} a=2 \\ a=\frac{-1}{2}\ \times \end{cases}$$

$$a=2\longrightarrow \lim_{x\to \frac{1}{2}} f(x)=\lim_{x\to \frac{1}{2}}\frac{x^2+x}{x^2+2x}=\frac{\frac{1}{4}+\frac{1}{2}}{\frac{1}{4}+1}=\frac{\frac{3}{4}}{\frac{5}{4}}=\frac{3}{5}$$