«صحیح کل سرفصل» «تألیف شماره ۲» «دفتر دوازدهم A»

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① $\Delta=0 \to (m+3)^2-4\times 6\times \frac{m}{2} \Rightarrow m^2+6m+9-12m \Rightarrow m^2-6m+9 \to \boxed{m=3}$

$\sqrt{6x^2+6x+\frac{3}{2}} \to \sqrt{6(x^2+x+\frac{1}{4})} \to \sqrt{6(x+\frac{1}{2})^2} \to \sqrt{6}\,(x+\frac{1}{2}) \to \boxed{a=-\frac{1}{2}}$

$|2x^3+\frac{1}{4}| = 2|x^3+\frac{1}{8}| \qquad \lim_{x\to 1} \frac{\sqrt{6}\,|x+\frac{1}{2}|}{2|x+\frac{1}{2}|\,(x^2-\frac{1}{2}x+\frac{1}{4})} = \frac{\sqrt{6}}{2\times\frac{3}{4}} = \frac{2\sqrt{6}}{3}$

$\frac{2\tan b}{\sqrt{\frac{1}{2}}} = \frac{2\sqrt{6}}{3} : 6\tan b = 2\sqrt{6}\times\frac{1}{\sqrt{2}} \to \tan b = \frac{\sqrt{3}}{3} \to \boxed{b=\frac{\pi}{6}}$

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② [illegible] $\Rightarrow 1 \to 5-a+b=0 \to \boxed{a-b=5}$

$\lim_{x\to 1}(x^2+ax+b)=0 \to 1+a+b=0 \to \boxed{a+b=-1} \Rightarrow \left.\begin{matrix} a=2 \\ b=-3 \end{matrix}\right\} \to \left[\frac{-3-4}{3}\right] = \left[\frac{-7}{3}\right] = \boxed{-3}$

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③ $\to 1)\ \tan\frac{3\pi}{4} = -1 \qquad \to 3)\ \lim_{x\to 5^-} \frac{(x-1)(x+2)}{3(1-x)} = \boxed{-\frac{7}{3}} \quad \boxed{f(5)=10b}$

$\to 2)\ \lim_{x\to 1^+} \frac{(x-1)(x+2)}{a(1-x)} = -1 \to -\frac{3}{a} = -1 \to \boxed{a=3}$

$10b = -\frac{7}{3} \to \boxed{b=-\frac{7}{30}}$

$a\times b = -\frac{7}{30}\times 3 = -0/7$

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④ $a[x+1] + \underbrace{b[x+[a+1]]}$ $\to$

$a[x+1] + b[x] + b[a+1] \to \underbrace{a([x]+1)}_{a[x]+a} + b[x] + b[a] + b$

$\to (a+b)[x] + (a+b+b[a])$

$a+b=0 \to b=-a \to f(x) = -a[a] \to \frac{a[a]}{-a[a]}$ -1

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⑤ $b[x^2-ax]-2a \to \boxed{b=0} \to f(x) = -2a \to \frac{a}{f(b)} = \frac{a}{-2a} = \boxed{-\frac{1}{2}}$

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6) $f(x) = \begin{cases} \dfrac{x^2+mx+n}{a-x} & a \neq x \\ 2 \end{cases}$

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$f(2a)=0 \rightarrow x=2a \rightarrow$ ریشه صورت

$x=a \rightarrow$ ریشه صورت + ریشه مخرج

$a=x \quad f(x) = \dfrac{(x-a)(x-2a)}{-(x-a)} = -(x-2a) \rightarrow f(x) = \begin{cases} -(x-2a) & x\neq a \\ 2 & x=a \end{cases}$

$\rightarrow \dfrac{4a^2+2am+n}{-a} = 0 \rightarrow 4a^2+2am+n=0$

$f(a) = -(a-2a) = 2 \rightarrow a=2$

$f(x) = \dfrac{(x-a)(x-2a)}{-(x-a)} = \dfrac{(x-2)(x-4)}{-(x-2)} = \dfrac{x^2-6x+8}{-(x-2)} = \dfrac{x^2+mx+n}{-(x-2)}$

$m=-6,\ n=8 \rightarrow n-m = 8-(-6) = 14$

7) $f(x) = \begin{cases} (1-a)[x] + (3a^2-1)[-x] & x \notin \mathbb{Z} \\ b \sin \frac{\pi}{a} & x \in \mathbb{Z} \end{cases}$

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راست: $(1-a)[Z^+] + (3a^2-1)[-Z^+] = (1-a)Z + (3a^2-1)(-Z-1)$ (I)

چپ: $(1-a)[Z^-] + (3a^2-1)[-Z^-] = (1-a)(Z-1) + (3a^2-1)(-Z)$ (II)

$I, II \rightarrow \lim_{x\to Z^+} f(x) = \lim_{x\to Z^-} f(x) \rightarrow -1+a = -3a^2+1 \rightarrow 3a^2+a-2=0 \rightarrow \begin{cases} a=-1 & \times \\ a=\frac{2}{3} \end{cases}$

$a=\frac{2}{3} \rightarrow b\sin\left(\frac{\pi}{\frac{2}{3}}\right) = -b \rightarrow -b = -\frac{1}{3} \rightarrow b=\frac{1}{3}$

$\frac{a}{b} = \dfrac{\frac{2}{3}}{\frac{1}{3}} = 2$

8) $f(x) = \begin{cases} \dfrac{|x^2+mx-m-1|}{m|x^2-1|+|x-1|} & x\neq 1 \\ \dfrac{m}{m+2} \end{cases}$

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$x \neq 1 \rightarrow \dfrac{x^2+mx-m-1}{mx^2-m+x-1}$

$m=4 \rightarrow \dfrac{|x^2+4x-5|}{4|x-1||x+1|+|x-1|} = \dfrac{|x-1||x+5|}{|x-1|(4|x+1|+1)} \rightarrow \lim_{x\to \frac{1}{4}} \dfrac{5+\frac{1}{4}}{4\times\frac{5}{4}+1} = \dfrac{\frac{21}{4}}{6} = \dfrac{7}{8}$

$x=1$

$\lim_{x\to \frac{1}{m}} \dfrac{|(x-1)(x+m+1)|}{|(x-1)(x+1)|+|x-1|} = \dfrac{|x-1||(x+m+1)|}{|x-1|(m|x+1|+1)} = \dfrac{|m+2|}{2m+1} = \dfrac{m}{m+2}$

$m \geq -2 \rightarrow m^2+4m+4 = 2m^2+m \rightarrow m^2-3m-4=0 \rightarrow \begin{cases} m=-1 \\ m=4 \end{cases}$

$a^2+a+m-2=0$

$a^2+ma-2a+a^2=0 \rightarrow$

$m=-1 \rightarrow \dfrac{|x^2-x|}{-|x-1||x+1|+|x-1|} = \dfrac{|x||x-1|}{|x-1|(-|x+1|+1)} \quad \times$

9) $f(x) = \dfrac{\sqrt{3}\,|ax+a|}{|x^3+(m-2)x+a^2|}$

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$x=a \rightarrow$ ریشه صورت و مخرج

$a^2+a=0 \rightarrow a(a+1)=0 \rightarrow \begin{cases} a=0 & \times \\ a=-1 & ✓ \end{cases}$

$a^3+a(m-2)+a^2=0 \xrightarrow{a=-1} -1-m+2+1=0 \rightarrow m=2$

$\lim_{x\to -1} \dfrac{\sqrt{3}|-x-1|}{|x^3+1|} = \dfrac{\sqrt{3}\,|-(x+1)|}{|(x+1)(x^2-x+1)|} = \dfrac{\sqrt{3}}{3}$

10) $f(x) = \begin{cases} \dfrac{x^2+|x|}{x^2+a|x|} & x \neq 0 \rightarrow \dfrac{x(x+1)}{x(x+a)} = \dfrac{1}{a} \\ \dfrac{2a-1}{2a+2} & x=0 \end{cases}$

$\Rightarrow \dfrac{1}{a} = \dfrac{2a-1}{2a+2} \rightarrow 2a^2-a = 2a+2 \rightarrow 2a^2-4a-2=0$

$\rightarrow a^2-4a-4=0 \rightarrow (a-4)(a+1)=0 \rightarrow a=-\frac{1}{2}$ XXX یا $a=2$ ✓✓✓

$\lim_{x\to\frac{1}{2}} \dfrac{\frac{1}{4}+\frac{1}{2}}{\frac{1}{4}+1} \rightarrow \dfrac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5} = 0.6$

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