نگین نکویی — تکلیف شماره ۲۱ — دوازدهم ریاضی A

۱- صورت و مخرج باید صفر باشد

$$f(x)=\begin{cases}\dfrac{\sqrt{6x^2+(m+3)x+\frac{m}{2}}}{|2x^3+(m-3)x^2+a^2|} & x\neq a\\[2ex] \dfrac{2\tan b}{\sqrt{-x}} & x=a\end{cases}$$

$2a^3+(m-3)a^2+a^2=0$

$2a^3+ma^2-2a^2=0$

$2a^3=2a^2-ma^2$) $a\neq 0$ چون در مخرج است

$2a=2-m$

$m=2-2a$

$$6a^2+(5-2a)a+\frac{2-2a}{2}=0$$

$6a^2+5a-2a^2+1-a=0 \rightarrow 4a^2+4a+1=0$

$a^2+4a+4=0$

$(a+2)^2=0 \rightarrow a=-2 \xrightarrow{\text{تقسیم}}_{\div 4} a=\frac{-2}{4}=-\frac{1}{2}$

$m=3$

$$\Rightarrow \frac{\sqrt{6x^2+\cancel{6}x+\frac{3}{2}}}{|2x^3+\frac{1}{4}|}=\frac{\sqrt{\frac{3}{2}}\,(|2x+1|)^2}{2\,|\cancel{2x+\frac{1}{2}}|\,|x^2-\frac{x}{2}+\frac{1}{4}|}=\frac{\sqrt{\frac{3}{2}}}{|x^2-\frac{x}{2}+\frac{1}{4}|}=\frac{2\tan b}{\sqrt{x}}\xrightarrow{x=-\frac{1}{2}}$$

$$\frac{\sqrt{\frac{3}{2}}}{\frac{3}{4}}=\frac{2\tan b}{\sqrt{\frac{1}{2}}}\Rightarrow \tan b=\frac{\sqrt{3}}{3}$$

$$b=\frac{\pi}{6}$$

۲- $5x^2-ax+b=0 \xrightarrow{x=1} 5-a+b=0$

$(a+2)^2=0 \to a=-2 \longrightarrow a=\frac{-2}{4}=-\frac{1}{2}$

$$\Rightarrow \frac{\sqrt{4x^2+\cancel{4}x+\frac{3}{4}}}{|2x^3+\frac{1}{4}|}=\frac{\sqrt{\frac{3}{4}}\,\cancel{|2x+1|}}{\cancel{2|x+\frac{1}{2}|}\,|x^2-\frac{x}{2}+\frac{1}{4}|}=\frac{\sqrt{\frac{3}{4}}}{|x^2-\frac{x}{2}+\frac{1}{4}|}=\frac{2\tan b}{\sqrt{x}} \xrightarrow{x=\frac{1}{2}}$$

$$\frac{\sqrt{\frac{3}{4}}}{\frac{3}{4}}=\frac{2\tan b}{\sqrt{\frac{1}{2}}} \Rightarrow \tan b=\frac{\sqrt{3}}{3}$$

$b=\frac{\pi}{6}$

۲ـ

$5x^2-ax+b=0 \xrightarrow{x=1} 5-a+b=0$

$a-b=5$

$\lim \frac{x^2+ax+b}{x-1}$

$1+a+b=0$

$a+b=-1$

$a-b=5$

$a=2 \to b=-3$

۲

$[\frac{b-2a}{2}]=[\frac{-3-4}{2}]=$ -4

۳ـ در نقاط مرزی حل می‌کنیم

۱٫۵

$$f(x)\begin{cases}\tan\frac{(2x+1)\pi}{4} & x<1\\ \frac{|x^2+x-2|}{a(1-x)} & 1<x<5\\ b(x-[-x]) & x\geq 5\end{cases}$$

$x<1 \to \tan\frac{3\pi}{4}=-1$

$\frac{|(x+2)(x-1)|}{a(1-x)}=\frac{(x+2)\cancel{(x-1)}}{a\cancel{(1-x)}}=-x=-1$

$a=3$

$+ \quad -2 \quad 1 \quad +$

$\xrightarrow{x=5} \frac{(x+2)\cancel{(x-1)}}{3\cancel{(1-x)}}=\frac{-1(7)}{3}$

$\xrightarrow{x\to5^+} b(x-(-6))$

$b(x+6)=-\frac{7}{3} \to 11b=-\frac{7}{3}$

$b=-\frac{7}{33}$

$ab=-\frac{7}{33}\times-\frac{7}{3}=\frac{49}{99}$

$ab=\frac{-7}{10}$

$f(5)=b(5-[-5])=b(5+5)=10b$

$$\lim_{x\to5^-}f(x)=\lim_{x\to5^-}\frac{|x^2+x-2|}{3(1-x)}=\lim_{x\to5^-}\frac{|25+5-2|}{3(1-5)}=\frac{-28}{12}=\frac{-7}{3}$$

$10b=\frac{-7}{3}$

$\boxed{b=\frac{-7}{30}}$

۴ـ

$a[x+1] + b[x+[a+1]]$

$= a[x] + a + b[x] + b[a] + b \xrightarrow{x \to 0^+} a + \cancel{b[a]} + \cancel{b}$

$\xrightarrow{x \to 0^-} \cancel{-a} + \cancel{a} - b + \cancel{b[a]} + \cancel{b}$

$a = -b$

$-a = b$

۲

$$\frac{a[a]}{f(a)} = \frac{a[a]}{\cancel{a[a]} + \cancel{a} + \cancel{a[a]} + a[a] + \cancel{a}} = \frac{a[a]}{-a[a]} = -1$$

۵ـ برای آنکه پیوسته باشد باید آن صفر باشد.

$f(x) = b[x^2 - ax] - 2a$ $\quad f(a) = -2a$

$\Rightarrow b = 0$

۲

$$\frac{a}{f(b)} = \frac{a}{-2a} = -\frac{1}{2}$$

۶ـ

$$f(x) = \begin{cases} \dfrac{x^2 + mx + n}{a - x} & x \neq a \\ 2 & x = a \end{cases}$$

$\longrightarrow$ مخرج $= 0$ ، صورت $= 0$

$a^2 + ma + n = 0$

$\Rightarrow \text{HOP} \Rightarrow \dfrac{2x + m}{-1} = \dfrac{2a + m}{-1} = 2$

$2a + m = -2$

$m = -2 - 2a$

$a^2 + ma + n = 0$

آن صفر با یک

$\Rightarrow b=0$

$$\frac{a}{f(0)} = \frac{a}{-2a} = -\frac{1}{2}$$

۶-

$$f(x)=\begin{cases} \dfrac{x^2+mx+n}{a-x} & x\neq a \\ 2 & x=a \end{cases} \longrightarrow \begin{matrix} \text{مخرج}=0 \\ \text{صورت}=0 \end{matrix} \quad a^2+ma+n=0$$

$$\Rightarrow \text{HOP} \Rightarrow \frac{2x+m}{-1} = \frac{2a+m}{-1}=2$$

$$2a+m=-2$$

$$m=-2-2a$$

$f(2a)=0$

$$a^2+ma+n=0$$

$$a^2+(-2-2a)(a)+n=0$$

$$a^2-2a-2a^2+n=0$$

$$-a^2-2a+n=0$$

$n-m=?\quad 8-(-6)=14$

$$f(2a) \Rightarrow \frac{4a^2+m(2a)+n}{a-2a} = \frac{4a^2+(-2-2a)(2a)+n}{a-2a}=0$$

$$-a^2+2a=0$$

$$a(-a+2)=0$$

غیرقابل قبول

$a=2$

2

$\rightarrow$ صورت $=0$

$$\cancel{4a^2}-4a-\cancel{4a^2}+n$$

$$-4a+n=0$$

$$n=4a$$

$$m=-2-2(2)$$

$$m=-6$$

$$n=8$$

۷-

$$f(x)=\begin{cases} (1-a)[x]+(3a^2-1)[-x] & x\notin \mathbb{Z} \\ b\sin\left(\frac{\pi}{a}\right) & x\in \mathbb{Z} \end{cases}$$

$$1-3a^2=-1+a$$

$$3a^2+a-2=0$$

$$a^2+a-6=0$$

$$(a+3)(a-2)=0$$

$a=-\frac{3}{2}$ غیرقابل قبول ، $a=\frac{2}{3}$

$$x\to 0^+ \quad (1-a)(0)+(3a^2-1)(-1)=1-3a^2$$

$$x\to 0^- \quad (1-a)(-1)+(3a^2-1)(0)$$

$$-1+a$$

$$x\to 0 \Rightarrow b\sin\left(\frac{\pi}{a}\right)$$

$a = \frac{3}{4}$ دور منفی $1 - 3a^2 = 1 - \frac{4}{3} = -\frac{1}{3}$

$a - 1 = -\frac{1}{4}$

$b \sin(\frac{3\pi}{4}) = -b \rightarrow b = \frac{1}{4}$

$\frac{a}{b} = \frac{\frac{3}{4}}{\frac{1}{4}} = 3$

2

8-

$$f(x) = \begin{cases} \dfrac{|x^2 + mx - m - 1|}{m|x^2-1| + |x-1|} & x \neq 1 \\ \dfrac{m}{m+2} & x = 1 \end{cases}$$

$\frac{\cancel{(x-1)}\,(x+m+1)}{|x-1|\,(m|x+1|)+1} = \frac{|m+2|}{2m+1} = \frac{m}{m+2}$

1.5

$\rightsquigarrow m<-2 \rightarrow -m^2 - 4m - 4 = 2m^2 + m$

$3m^2 + 5m + 4 = 0 \rightarrow$ غیر قابل قبول $\Delta < 0$

$\rightarrow m > -2 \quad m^2 + 4m + 4 = 2m^2 + m$

$m^2 - 3m - 4 = 0$

$(m-4)(m+1) = 0$

$m = 4 \qquad m = -1$

$m = 4 \rightarrow \frac{|x^2 + 4x - 5|}{4|x-1||x+1| + |x-1|} = \frac{|x-1||x+5|}{|x-1|(4|x+1|+1)} \rightarrow$

$\lim_{x\to\frac{1}{4}} \frac{5+\frac{1}{4}}{4\times\frac{5}{4}+1} = \frac{\frac{21}{4}}{6} = \frac{7}{8}$

$\lim_{x\to\frac{1}{2}} f(x) = \frac{|\frac{1}{4}+\frac{1}{4}+1|}{(\frac{1}{2}\times\frac{5}{4})+1} = \frac{\frac{3}{2}}{\frac{19}{4}} = \frac{24}{19}$

$\frac{|-1-1+1|}{1} = 1$

9-

$f(x) = \sqrt{3} \quad a|x+1|$

$R - \{a\}$ $\rightarrow$ ریشه مخرج a

$a = \frac{2}{3}$ جا جواب $\quad 1 - 2a^2 = \ldots = \frac{1}{9}$ $\qquad \frac{a}{b} = \frac{\frac{2}{3}}{\frac{1}{3}} = 2$

$a - 1 = -\frac{1}{3}$

$b \sin\left(\frac{3\pi}{2}\right) = \ldots \Rightarrow b = \frac{1}{3}$

$$f(x) = \begin{cases} \dfrac{|x^2 + mx - m - 1|}{m|x^2 - 1| + |x - 1|} & x \neq 1 \\[2ex] \dfrac{m}{m+2} & x = 1 \end{cases}$$

$\dfrac{\cancel{(x-1)}(x+m+1)}{\cancel{|x-1|}(m(x+1)) + 1} = \dfrac{|m+2|}{2m+1} = \dfrac{m}{m+2}$

اگر $m < -2 \Rightarrow -m^2 - 4m - 4 = 2m^2 + m$

$3m^2 + 5m + 4 = 0 \Rightarrow$ جواب ندارد

$\rightarrow m \geq -2 \quad m^2 + 4m + 4 = 2m^2 + m$

$m^2 - 3m - 4 = 0$

$(m-4)(m+1) = 0$

$m = 4 \qquad m = -1$

$\lim\limits_{x \to \frac{1}{2}} f(x) = \dfrac{\left(\frac{1}{4} + \frac{1}{2} + 1\right)}{\left(\frac{1}{2} \times \frac{9}{2}\right) + 1} = \dfrac{\frac{3}{4}}{\frac{19}{4}} = \dfrac{24}{19} \qquad \dfrac{|-1-1+1|}{1} = 1$

9-

$f(x) = \dfrac{\sqrt{3}\, a|x+1|}{|ax^2 + (a-2)x + a^2|} \qquad R - \{a\}$ $\rightarrow$ a ریشه مخرج و صورت است.

$\frac{m}{m+2}$ $\qquad x:1$

$\leadsto m<-2 \to -m^2-4m-4=2m^2+m$

$3m^2+5m+4=0 \to$ غیر ممکن $\Delta<0$

$\hookrightarrow m>-2 \quad m^2+4m+4=2m^2+m$

$m^2-3m-4=0$

$(m-4)(m+1)=0$

$m=4 \qquad m=-1$

$$\lim_{x\to\frac{1}{2}} f(x)=\frac{\left|\frac{1}{4}+\frac{1}{4}+1\right|}{\left(\frac{1}{2}\times\frac{5}{2}\right)+1}=\frac{\frac{3}{2}}{\frac{19}{4}}=\frac{24}{19} \qquad \frac{|-1-1+1|}{1}=1$$

-9

$$f(x)=\frac{\sqrt{3}\;a|x+1|}{|x^3+(m-2)x+a^2|} \qquad R-\{a\}$$

$\to$ a ریشه مضاعف مخرج است.

$a(a+1)=0 \to a=0$ صورت $=0$ $f(x)=0$ غ ق

$\hookrightarrow a=-1$ $\quad |-1+(m-2)(-1)+1|=0$

$-m+2=0$

$m=2$

$$\to f(x)=\frac{\sqrt{3}\,|-x-1|}{|x^3-x+1|}=\frac{0}{0}$$

$$\to \frac{\sqrt{3}\,\cancel{|x+1|}}{\cancel{|x+1|}\,|x^2-x+1|}=\frac{\sqrt{3}}{3}$$

2

-10

$$f(x)=\begin{cases}\dfrac{x^2+|x|}{x^2+a|x|}\\[2mm] \dfrac{2a-1}{2a+2}\end{cases}=\frac{|x|(|x|+1)}{|x|(|x|+a)}=\frac{|x|+1}{|x|+a}$$

$$\frac{1}{a}=\frac{2a-1}{2a+2}$$

$2a^2-a=2a+2$

$2a^2-3a-2=0$

$a^2-3a-4=0$

$(a-4)(a+1)=0$

$4 \checkmark \quad \cancel{-1}$

$$\lim_{x\to\frac{1}{2}} f(x)=\frac{\frac{1}{2}+1}{\frac{1}{2}+2}=\frac{\frac{3}{2}}{\frac{5}{2}}=\frac{3}{5}$$

$$\lim_{x\to-2} f(x)=\frac{2+1}{2-\frac{1}{2}}=2$$ غ ق

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