۱۵/۲۵

Date: Sub: مهتا محلوجی - پاسخ تکلیف

۱-
$$f(x)=\begin{cases}\dfrac{\sqrt{6x^2+(m+3)x+\frac{x}{2}}}{|2x^3+(m-3)x^2+a^2|} & x\neq a \\ \dfrac{2\tan b}{\sqrt{-x}} \quad * a<0 & x=a\end{cases}$$

$* \ 0<b<\pi$

$\Delta=0 \rightarrow (m+3)^2-12m=0 \rightarrow (m-3)^2=0 \rightarrow m=3$

$x=a=\dfrac{-(m+3)}{12} \rightarrow a=-\dfrac{1}{2}$

$\lim\limits_{x\to-\frac{1}{2}}\dfrac{\sqrt{6(x+\frac{1}{2})^2}}{|2x^3+\frac{1}{4}|}=\lim\limits_{x\to-\frac{1}{2}}\dfrac{\sqrt{6}\,|x+\frac{1}{2}|}{2|x^3+\frac{1}{8}|} \Rightarrow$

$2|x+\frac{1}{2}||x^2-\frac{x}{2}+\frac{1}{4}|$

$\dfrac{\sqrt{6}}{\frac{3}{2}}=\dfrac{2\sqrt{6}}{3}$

$f(-\frac{1}{2})=\dfrac{2\tan b}{\sqrt{\frac{1}{2}}}=\dfrac{2\sqrt{6}}{3} \Rightarrow \sqrt{2}\tan b=\dfrac{\sqrt{6}}{3}$

$b=\dfrac{\pi}{6}$ ✓

(۲)

۲- $5x^2-ax+b=0 \rightarrow f(x)=\dfrac{x^2+ax+b}{x-1}$

f پیوستگی را صددارد $\leftarrow x=1$ ریشه صورت

$5-a+b=0$

$*\ a-b=5$

$1+a+b=0$

$*\ a+b=-1$

$a-b=5$
$a+b=-1$
$2a=4 \rightarrow a=2 \quad b=-3$

$\left[\dfrac{b-2a}{3}\right]=?$

$\left[\dfrac{-3-4}{3}\right]=-3$ ✓

(۲)

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$\tan\frac{3\pi}{4} = -1$ ($x=1$)

$$f(x) = \begin{cases} \tan\frac{(2x+1)\pi}{4} & x \leq 1 \\ \dfrac{-(x+2)(x-1)\,|x^2+x-2|}{a(1-x)} & 1 < x < 5 \\ b\,(x-[-x]) & x \geq 5 \end{cases}$$

۳- f روی بازه‌ی $[1,5]$ پیوسته $ab=?$

$-2 \quad 1$; $+ \quad - \quad +$

$\rightarrow \dfrac{-(x+2)}{a}$

$b(x-[-x]) \rightarrow 10b$

(۲)

$f(1) = \lim\limits_{x\to 1^+} f(x) \Rightarrow -1 = -\dfrac{(x+2)}{a} \Rightarrow a=3$

$f(5) = \lim\limits_{x\to 5^-} f(x) \rightarrow 10b = \dfrac{-7}{a}_{3} \rightarrow b = -\dfrac{7}{30}$

$ab = -0.7$

۴- $f(x) = a[x+1] + b[x+[a+1]]$ در $\mathbb{R}$ پیوسته ، $\dfrac{a[a]}{f(a)}$

$[a+1] = [a]+1$

$a[x]+a+b[x]+b[a]+b$

$[x](a+b)+b[a]+a+b$

ضرب عامل صفرشونده باعث پیوستگی $\rightarrow a+b=0 \quad a=-b \rightarrow f(a) = -a[a]$

$\Rightarrow \dfrac{a[a]}{-a[a]} = -1$

(۲)

۵- $f(x) = b[x^2-ax]-2a$ پیوسته در $\mathbb{R}$ ، $\dfrac{a}{f(a)} = ?$

عامل صفرشونده (b)

$b=0$

$f(a) = -2a \rightarrow \dfrac{a}{f(a)} = \dfrac{a}{-2a} = -\dfrac{1}{2}$

(۲)

$$f(x) = \begin{cases} \dfrac{x^2 + mx + n}{a - x} & x \neq a \\ 2 & x = a \end{cases}$$

-6

$f(2a) = 0$

$n - m = ?$

$$\lim_{x \to a} \frac{x^2 + mx + n}{a - x} = 2 \xrightarrow{hop} \frac{2x + m}{-1} = 2$$

$2a + m = -2$

1,25

به‌ازای $x = a \xrightarrow{*} a^2 + ma + n = 0$

$$f(2a) = 0 \longrightarrow \frac{4a^2 + 2am + n}{a} = 0 \longrightarrow 4a^2 + 2am + n = 0$$

$$4a^2 + 2am + n = a^2 + ma + n \longrightarrow 3a^2 = -2am$$

$3a = -2m$

$-\frac{3}{2} a = m$

$2a - \frac{3}{2} a$

$\frac{1}{2} a = -2 \longrightarrow a = -4$

$m = \frac{3}{2} \times 4 = 6$

$m = 6$

$m = -6$

$* \; 16 + (-24) + n = 0$

-8

$n = 8$

$n - m = 8 - 6 = 2$

$n - m = 8 - (-6) = 14$

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۷-

$$f(x)=\begin{cases}(1-a)[x]+(3a^2-1)[-x] & x\notin \mathbb{Z}\\ b\sin(\frac{\pi}{a}) & x\in\mathbb{Z}\end{cases}$$

f روی R پیوسته

$\frac{a}{b}=?$

$1-a=3a^2-1 \to 3a^2+a-2=0$

$a=-1 \quad a=\frac{2}{3}$

۲

$2([x]+[-x])$

$\xrightarrow{x\in\mathbb{Z}} 0$

$b\sin(-\pi)=0$

$b=1$ (crossed out)

$\frac{1}{3}([x]+[-x])=0$

$b\sin(\frac{3\pi}{2})=\frac{1}{3} \to b=\frac{1}{3}$ ($\frac{1}{3}$ crossed out; corrected to $-\frac{1}{3}$)

$\frac{a}{b}=2$ ✓

۸-

$$\lim_{x\to\frac{1}{m}}\frac{|(x-1)(x+m+1)|}{|(x-1)(x+1)|+|x-1|}=\frac{|x-1||(x+m+1)|}{|x-1|(m|x+1|+1)}=\frac{|m+2|}{2m+1}=\frac{m}{m+2}$$

$m\geq -2 \to m^2+4m+4=2m^2+m \to m^2-3m-4=0 \to \begin{cases}m=-1\\ m=4\end{cases}$

$$m=4\to \frac{|x^2+4x-5|}{4|x-1||x+1|+|x-1|}=\frac{|x-1||x+5|}{|x-1|(4|x+1|+1)}\to \lim_{x\to\frac{1}{4}}\frac{5+\frac{1}{4}}{4\times\frac{5}{4}+1}=\frac{\frac{21}{4}}{6}=\frac{7}{8}$$

$$m=-1\to \frac{|x^2-x|}{-|x-1||x+1|+|x-1|}=\frac{|x||x-1|}{|x-1|(-|x+1|+1)}$$ ✗

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۹- ⓪

$x=a \to$ ریشه صورت و مخرج

$a^2+a=0 \to a(a+1)=0 \to \begin{cases} a=0 \ ✗ \\ a=-1 \ ✓ \end{cases}$

$a^3+a(m-2)+a^2=0 \xrightarrow{a=-1} -1-m+2+1=0 \to m=2$

$\lim_{x\to -1} \frac{\sqrt{3}\,|-x-1|}{|x^3+1|} = \frac{\sqrt{3}\,|-(x+1)|}{|(x+1)(x^2-x+1)|} = \frac{\sqrt{3}}{3}$

۱۰-

$f(x) = \begin{cases} \dfrac{x^2+|x|}{x^2+a|x|} & x \neq 0 \\ \dfrac{2a-1}{2a+2} & x=0 \end{cases}$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x^2+x}{x^2+ax} \quad \frac{0}{0} \xrightarrow{Hop} \lim_{x\to 0^+} \frac{2x+1}{2x+a} \to \frac{1}{a}$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x^2-x}{x^2-ax} \xrightarrow[Hop]{\frac{0}{0}} \lim \frac{2x-1}{2x-a} = \frac{1}{a}$

$f(x) = \frac{2a-1}{2a+1} = \frac{1}{a} \to 2a^2-a = 2a+2$

$2a^2-3a-2=0 \to a=-\frac{1}{2}$ و 2

$\lim_{x\to \frac{1}{a}} f(x) = \xrightarrow{a=2} \frac{0}{0}$ ✓ ②

~~$a=-\frac{1}{2}$~~