Date: Sub: مهسا محمودی - ریاضی تکلیف

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$$f(x)=\begin{cases}\dfrac{\sqrt{6x^2+(m+3)x+\frac{m}{2}}}{|2x^3+(m-3)x^2+a^2|} & x\neq a\\[2ex] \dfrac{2\tan b}{\sqrt{-x}} \quad *a<0 & x=a\end{cases}$$

$* \; 0<b<\pi$

$\Delta=0 \rightarrow (m+3)^2-12m=0 \rightarrow (m-3)^2=0 \rightarrow m=3$

$x=a=\dfrac{-(\overset{3}{m}+3)}{12} \longrightarrow a=-\frac{1}{2}$

$$\lim_{x\to -\frac{1}{2}} \frac{\sqrt{6(x+\frac{1}{2})^2}}{|2x^3+\frac{1}{4}|}=\lim_{x\to -\frac{1}{2}}\frac{\sqrt{6}\,\cancel{|x+\frac{1}{2}|}}{2|x^3+\frac{1}{8}|} \Rightarrow$$

$2\cancel{|x+\frac{1}{2}|}\,|x^2-\frac{x}{2}+\frac{1}{4}|$

$\dfrac{\sqrt{6}}{\frac{3}{2}}=\dfrac{2\sqrt{6}}{3}$

$\hookrightarrow f(-\frac{1}{2})=\dfrac{2\tan b}{\sqrt{\frac{1}{2}}}=\dfrac{2\sqrt{6}}{3} \Rightarrow \sqrt{2}\tan b=\dfrac{\sqrt{6}}{3}$

$b=\frac{\pi}{6}$

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$5x^2-ax+b=0 \rightarrow f(x)=\dfrac{x^2+ax+b}{x-1}$

f ناپیوستگی رفع‌شدنی دارد $\longleftarrow$ $x=1$ ریشه صورت

$5-a+b=0$

$* \; a-b=5$

$1+a+b=0$

$* \; a+b=-1$

$$\begin{array}{l} a-b=5\\ a+b=-1\\ \hline 2a=4 \rightarrow a=2 \quad b=-3\end{array}$$

$\left[\dfrac{b-2a}{3}\right]=?$

$\left[\dfrac{-3-4}{3}\right]=-3$

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$\tan\frac{3\pi}{4} = -1$

$x=1$

$$f(x) = \begin{cases} \tan\frac{(2x+1)\pi}{4} & x \leq 1 \\ \dfrac{-(x+2)(x-1)}{\dfrac{|x^2+x-2|}{a(1-x)}} \to \dfrac{-(x+2)}{a} & 1 < x < 5 \\ b(x-[-x]) \to 10b & x \geq 5 \end{cases}$$

$-2 \quad 1$

$+ \circ - \circ +$

-۳ f روی بازه ی $[1, 5]$ پیوسته $ab = ?$

$f(1) = \lim_{x \to 1^+} f(x) \Rightarrow -1 = -\frac{(x+2)}{a} \Rightarrow a = 3$

$f(5) = \lim_{x \to 5^-} f(x) \to 10b = \frac{-7}{a} \to b = \frac{-7}{30}$

$ab = -0.7$

-۴ $f(x) = a[x+1] + b[x + \overbrace{[a+1]}^{[a]+1}] \longrightarrow$ در $\mathbb{R}$ پیوسته $\quad \frac{a[a]}{f(x)}$

$a[x] + a + b[x] + b[a] + b$

$[x](a+b) + b[a] + a + b \Rightarrow \frac{a[a]}{-a[a]} = -1$

ضرب عامل صفر شونده باعث پیوستگی $\longrightarrow a+b = 0 \longrightarrow f(a) = -a[a]$

$a = -b$

-۵ $f(x) = b[x^2 - ax] - 2a$ پیوسته در $\mathbb{R}$ $\quad \frac{a}{f(a)} = ?$

عامل صفر شونده

$b = 0 \qquad f(a) = -2a \to \frac{a}{f(a)} = \frac{a}{-2a} = -\frac{1}{2}$

$$f(x)=\begin{cases}\dfrac{x^2+mx+n}{a-x} & x\neq a\\[2mm] 2 & x=a\end{cases}$$

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$f(2a)=0$

$n-m=?$

$$\lim_{x\to a}\frac{x^2+mx+n}{a-x}=2 \xrightarrow{\text{hop}} \frac{2x+m}{-1}=2$$

$$2a+m=-2$$

ریشه است $x=a \xrightarrow{*} a^2+ma+n=0$

$$f(2a)=0 \longrightarrow \frac{4a^2+2am+n}{a}=0 \longrightarrow 4a^2+2am+n=0$$

$$4a^2+2am\cancel{+n}=a^2+ma\cancel{+n} \longrightarrow 3a^2=-2am$$

$$3a=-2m$$

$$-\frac{3}{2}a=m$$

$$*\ \underbrace{16+(-24)}_{-8}+n=0$$

$$\boxed{n=8}$$

$$2a-\frac{3}{2}a$$

$$\frac{1}{2}a=-2 \longrightarrow a=-4$$

$$m=\frac{3}{2}\times 4=6$$

$$m=6$$

$$n-m=8-6=2$$

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$$f(x) = \begin{cases} (1-a)[x] + (3a^2-1)[-x] & x \notin \mathbb{Z} \\ b\sin\left(\frac{\pi}{a}\right) & x \in \mathbb{Z} \end{cases}$$

f روی R پیوسته

$\frac{a}{b} = ?$

$1 - a = 3a^2 - 1 \rightarrow 3a^2 + a - 2 = 0$

$a = -1 \quad a = \frac{2}{3}$

$2([x] + [-x])$

$\xrightarrow{x \in \mathbb{Z}} 0$

$b\sin(\underbrace{-\pi}_{0}) = 0$

$b = \cancel{1}$

$\frac{1}{3}([x] + [-x]) = 0$

$b\sin\left(\underbrace{\frac{3\pi}{2}}_{-1}\right) = -\frac{1}{3} \rightarrow b = \frac{1}{3}$

$\frac{a}{b} = 2$

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$$f(x)=\begin{cases}\dfrac{x^2+|x|}{x^2+a|x|} & x\neq 0\\[2ex] \dfrac{2a-1}{2a+2} & x=0\end{cases}$$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{x^2+x}{x^2+ax}\ \xrightarrow[\text{Hop}]{\frac{0}{0}}\ \lim_{x\to 0^+}\frac{2x+1}{2x+a}\to\frac{1}{a}$$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\frac{x^2-x}{x^2-ax}\ \xrightarrow[\text{Hop}]{\frac{0}{0}}\ \lim\frac{2x-1}{2x-a}=\frac{1}{a}$$

$$f(0)=\frac{2a-1}{2a+2}=\frac{1}{a}\longrightarrow 2a^2-a=2a+2$$

$$2a^2-3a-2=0\to a=-\frac{1}{2},\,2$$

$$\lim_{x\to\frac{1}{a}} f(x)= \begin{cases} a=2 \to \\ a=-\frac{1}{2} \to 2\end{cases}$$