۱-

$|2x^3+(m-3)x^2+a^2| \geq 0 \xrightarrow{x=a} 2a^3+(m-3)a^2+a^2=0 \to a^2(2a+(m-2))=0$

$\downarrow a=0 \qquad \downarrow 2a=2-m,\quad a=\frac{2-m}{2}, \quad m=2-2a$

$6x^2+(m+3)x+\frac{m}{2} \xrightarrow{x=a} 6a^2+(2-2a+3)a+\frac{2-2a}{2}$

$=4a^2+4a+1=0 \to (2a+1)^2=0 \to a=-\frac{1}{2} \qquad m=2+1=3$

$\frac{\sqrt{6x^2+(m+3)x+\frac{m}{2}}}{|2x^3+(m-3)x^2+a^2|} = \frac{\sqrt{6x^2+6x+\frac{3}{2}}}{|2x^3+\frac{1}{4}|} = \frac{\sqrt{\frac{3}{2}}\,|2x+1|}{|2x+1|\,|x^2-\frac{x}{2}+\frac{1}{4}|}$

$\to \frac{\sqrt{\frac{3}{2}}}{|x^2-\frac{x}{2}+\frac{1}{4}|} = \frac{2\tan b}{\sqrt{-x}} \xrightarrow{x=-\frac{1}{2}} \frac{\sqrt{\frac{3}{2}}}{\frac{3}{4}} = \frac{2\tan b}{\sqrt{\frac{1}{2}}} \to \tan b=\frac{\sqrt{3}}{3}$

$b=\frac{\pi}{6}$ ✓ (۲)

۲-

$f(x)=\frac{x^2+ax+b}{x-1} \to x=1 \to 1+a+b=0 \to a+b=-1$

$5x^2-ax+b=0 \xrightarrow{x=1} 5-a+b=0 \to a-b=5$

$\left.\begin{array}{l} a+b=-1 \\ a-b=5 \end{array}\right\} \to a=2,\ b=-3$

$\left[\frac{b-2a}{3}\right]=\left[\frac{-3-4}{3}\right]=-3$ ✓ (۲)

۳-

$f(x)=\begin{cases} \tan\frac{(x+1)\pi}{4} & x<1 \\ \frac{|x^2+x-1|}{a(1-x)} & 1<x<5 \\ b(x-[-x]) & x\geq 5 \end{cases}$

$x<1 \to x=1 \quad \tan\frac{3\pi}{4}=-1$

$1<x<5 \to \frac{(x+2)(x-1)}{a(1-x)} = \frac{-x-2}{a} \xrightarrow{x=1} \frac{-3}{a}=-1 \to a=3$

$x=5 \to \frac{-7}{3}$

$x\geq 5 \to x=5 \to b\times 10=\frac{-7}{3} \to b=\frac{-7}{30}$

$a\times b=\frac{-7}{30}\times 3=-\frac{7}{10}$ ✓ (۲)

۴-

$f(x)=a[x+1]+b[x+[a+1]]=a[x]+a+b[x]+b[a]+b$

$x=0^+ \to a+b[a]+b$

$x=0^- \to -a+a-b+b[a]+b$

$\} \to a+b=0 \to b=-a$

$f(a)=a[a]+a+b[a]+b[a]+b=b[a]=-a[a] \qquad \frac{a[a]}{-a[a]}=-1$ ✓ (۲)

Arman

۵- $f(x)=b[x^2-cx]-2a \Rightarrow b=0 \Rightarrow f(x)=-2a \qquad \frac{a}{f(x)}=\frac{a}{-2a}=-\frac{1}{2}$ ✓ ۲

۶- $\frac{(a-x)(2a-x)}{a-x}=2 \xrightarrow{x=a} a=2$

$(2-x)(4-x)=x^2-6x+8 \Rightarrow m=-6,\ n=8 \qquad n-m=8-(-6)=14$ ✓ ۲

۷- $x=0^- \Rightarrow f(x)=1-3a^2$

$x=0 \Rightarrow f(x)=b\sin(\frac{\pi}{a})$

$x=0^+ \Rightarrow f(x)=a-1$

$\Rightarrow 1-3a^2=a-1 \Rightarrow 3a^2+a-2=0 \Rightarrow a=-1 \Rightarrow b\sin(\frac{\pi}{a})=b\sin(-\pi)$

$a=\frac{2}{3} \Rightarrow b\sin(\frac{\pi}{a})=b\sin(\frac{3\pi}{2})$ ۲

$\Rightarrow a=-1 \Rightarrow b\sin(-\pi)=-2$ ✗ $\qquad a=\frac{2}{3} \Rightarrow b\sin(\frac{3\pi}{2})=-\frac{1}{3} \Rightarrow b=\frac{1}{3}$

$\frac{a}{b}=\frac{\frac{2}{3}}{\frac{1}{3}}=2$ ✓

۸- $\frac{|x+m+1|\,|x-1|}{m|x-1|\,|x+1|+|x-1|}=\frac{|x+m+1|}{m|x+1|+1}=\frac{m}{m+2} \xrightarrow{x=1} \frac{m+2}{2m+1}=\frac{m}{m+2}$

$\Rightarrow 2m^2+m=(m+2)^2 \Rightarrow m^2-3m-4=0 \Rightarrow m=4,\ -1$ غ ق ق ۲

$\lim_{x\to \frac{1}{4}} f(x)=\frac{|x+m+1|}{m|x+1|+1}=\frac{|\frac{1}{4}+5|}{4|\frac{5}{4}|+1}=\frac{21}{24}$ ✓

۹- $\sqrt{3}|a^2+a|=0 \Rightarrow a^2+a=0 \Rightarrow a=0,\ -1$ (غ ق ق) ۱٫۷۵

$(-1)^3+m-2+1=0 \Rightarrow m=2 \qquad \lim_{x\to -1} f(x)=\frac{\sqrt{3}|-x-1|}{|x^3+1|} \overset{\frac{0}{0}}{=} \frac{\sqrt{3}|x+1|}{|x+1|\,|x^2-x+1|}=\frac{\sqrt{3}}{3}$

$\lim_{x\to -1}\frac{\sqrt{3}|-x-1|}{|x^3+1|}=\frac{\sqrt{3}|-(x+1)|}{|(x+1)(x^2-x+1)|}=\frac{\sqrt{3}}{3}$

۱۰- $\frac{x(x+1)}{x(x+a)}=\frac{2a-1}{2a+2} \xrightarrow{x=0} \frac{2a-1}{2a+2}=\frac{1}{a} \Rightarrow 2a^2-3a-2=0 \Rightarrow a=2$

$a=-\frac{1}{2}$ غ ق ق

$\lim_{x\to \frac{1}{2}} f(x)=\frac{x^2+|x|}{x^2+2|x|}=\frac{\frac{3}{4}}{\frac{5}{4}}=\frac{3}{5}$ ✓ ۲