- بنام خدا -

۸

Subject — تکالیف برای سادات حسینی

Year Month Day ( ) page: ( )

۵

$$(f\circ g)'(-\sqrt[3]{2}) = g'(-\sqrt[3]{2}) \times f'(g(-\sqrt[3]{2})) \Rightarrow x = -\sqrt[3]{2}$$

$$\Rightarrow \begin{cases} f(x) = \dfrac{1}{\sqrt[3]{2x}} \\ g(x) = \dfrac{1}{2x^3} \end{cases}$$

۲

$$f(g(x)) = f\left(\frac{1}{2x^3}\right) = \frac{1}{\sqrt[3]{\frac{2}{2x^3}}} = x \Rightarrow (f\circ g)' = (x)' = 1$$

۷

$$g\left(\frac{3}{\sqrt{8}}\right) = \frac{1}{\sqrt[3]{\frac{9}{8}-1}} = 2 \Rightarrow (f\circ g)'\left(\frac{3}{\sqrt{8}}\right) = g'\left(\frac{3}{\sqrt{8}}\right) \times f'(2) \quad \mathrm{I}$$

$$g(x) = (x^2-1)^{-\frac{1}{3}} \Rightarrow g'(x) = -\frac{1}{3}(x^2-1)^{-\frac{4}{3}} \times 2x = \frac{-2x}{3\sqrt[3]{(x^2-1)^4}}$$

$$\Rightarrow g'\left(\frac{3}{\sqrt{8}}\right) = \frac{-2 \times \frac{3}{\sqrt{8}}}{3\sqrt[3]{\left(\frac{1}{8}\right)^4}} = \frac{-16}{\sqrt{2}} = -8\sqrt{2}$$

۲

$$f(x) = (4x)^2 + 1 \Rightarrow f'(x) = 32x \Rightarrow f'(2) = 64$$

$$\overset{\mathrm{I}}{\Rightarrow} (f\circ g)'\left(\frac{3}{\sqrt{8}}\right) = -8\sqrt{2} \times 64 = -128\sqrt{2} \times 4$$

سی ۴ برابر است!

۹

$$\lim_{h\to 0} \frac{f^2(5-h) - 3f(5-h) + 2}{h(5-h)} = \lim_{h\to 0} \frac{(f(5-h)-1)(f(5-h)-2)}{h(5-h)}$$

$$f(5) = 1 \times \sqrt[3]{8} = 2 \Rightarrow \lim \frac{f(5-h)-1}{5-h} \times \lim \frac{f(5-h)-2}{h} = \frac{f(5-h)-2}{h} \times \lim_{h\to 0} \frac{f(5-h)-2}{h}$$

برابر!

$$= \frac{2-1}{5} \times \lim_{h\to 0} \frac{f(5+(-h)) - f(5)}{h} = -\frac{1}{5} \lim_{h\to 0} \frac{f(5+(-h)) - f(5)}{(-h)} = -\frac{1}{5} f'(5)$$

$$f(x) = (x-4)\sqrt[3]{x+3} \Rightarrow f'(x) = 1 \times \sqrt[3]{x+3} + \frac{x-4}{3\sqrt[3]{(x+3)^2}} \Rightarrow f'(5) = \sqrt[3]{8} + \frac{1}{3\sqrt[3]{64}} = 2 + \frac{1}{12}$$

$$\Rightarrow \frac{25}{12} \Rightarrow -\frac{1}{5} f'(5) = -\frac{1}{5} \times \frac{25}{12} = -\frac{5}{12}$$

۲

به نام خدا

Subject: تکلیف پویا سادات حسینی

Year : Month : Day : ( ) page: ( )

۱۰

$4y = 3x + 1 \Rightarrow y = \frac{3}{4}x + \frac{1}{4} = \frac{1}{2}x + \frac{1}{4} \Rightarrow$ شیب $= \frac{1}{2}$

۲

شیب خط مماس $= -2$

$y = x^2 - 4x + 5$ مشتق $\Rightarrow y' = 2x - 4 = -2 \Rightarrow x = 1 \Rightarrow y = 1 - 4 + 5 = 2 \Rightarrow \left| \begin{array}{l} x = 1 \\ y = 2 \end{array} \right.$

سوال ۱ – تابع $|x|$ در $x=0$ نقطه گوشه دارد پس مشتق ناپذیر است.

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) \to a+1=b \to a-b=-1 \rightsquigarrow a-b=-1 \to a-\frac{3}{2}=-1 \to a=\frac{1}{2}$$

$$f'_+(1) = f'_-(1) \to 1 = \frac{2}{3} b \times x^{-\frac{1}{3}} \to 1 = \frac{2}{3} b (1)^{-\frac{1}{3}} \to b = \frac{3}{2} \qquad a+b = \frac{3}{2}+\frac{1}{2} = \frac{4}{2} = 2$$

سوال ۲

$$f(x) = \frac{|2x-4|}{\sqrt[3]{x^2}} \xrightarrow{x=1} \frac{-2x+4}{\sqrt[3]{x^2}}$$

$$f'(x) = \left(\frac{-2x+4}{\sqrt[3]{x^2}}\right)' = \frac{-2(1) - \frac{2}{3\sqrt[3]{x}}(2)}{\sqrt[3]{x^4}} \longrightarrow f'(1) = \frac{-2(1) - \frac{2}{3}(2)}{1} = \frac{-10}{3}$$

$$g(x) = \frac{\sqrt{x^2-4x+4} + \sqrt{3x}}{\sqrt[3]{x^2}} = \frac{|x-2| + \sqrt{3x}}{\sqrt[3]{x^2}} \xrightarrow{x=1} \frac{(-x+2)+\sqrt{3x}}{\sqrt[3]{x^2}}$$

$$g'(x) = \frac{\left(-1+\frac{3}{2\sqrt{3x}}\right)(1) - \frac{2}{3\sqrt[3]{x}}(1+\sqrt{3})}{\sqrt[3]{x^4}} \to g'(1) = \frac{\left(-1+\frac{3}{2\sqrt{3}}\right) - \frac{2}{3}(1+\sqrt{3})}{1} = -\frac{5}{3} - \frac{\sqrt{3}}{6}$$

$$f'(1) - 2g'(1) = \frac{-10}{3} - 2\left(\frac{-5}{3} - \frac{\sqrt{3}}{6}\right) = \frac{\sqrt{3}}{3}$$

سوال ۳

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) \longrightarrow \frac{1}{a} = ba+c$$

$$f'(x) = \begin{cases} b & x<a \\ \frac{-1}{x^2} & x \geq a \end{cases} \to f'_+(a) = f'_-(a) \to b = \frac{-1}{a^2}$$

$$\to \frac{1}{a} = \frac{-1}{a^2} \times a + c$$

$$\frac{2}{a} = c \qquad ac = 2$$

سوال ۴

نقطه گوشه $f \to f'_-(a) \neq f'_+(a)$

$m=0 \to f'_-(a) = f'_+(a)$ × غ ق ق

$$m=1 \to \begin{cases} f'_-(a) = 0 \\ f'_+(a) = 1 \end{cases} \to f'_-(a) \neq f'_+(a)$$

$$m \geq 2 \to \begin{cases} f'_-(a) = 0 \\ f'_+(a) = 0 \end{cases} \to f'_-(a) = f'_+(a)$$ غ ق ق

یک مقدار صحیح m

سوال ۶

$$f(3) = -3a-5 \quad , \quad f'(3) = -a$$

$$f'(3) - f(3) = 2 \longrightarrow (-a) - (-3a-5) = 2 \to a = \frac{-3}{2}$$

$$\frac{f(3)}{f'(3)} = \frac{-3a-5}{-a} = \frac{-3\left(-\frac{3}{2}\right)-5}{\frac{3}{2}} = \frac{-1}{3}$$

سوال ۸

$$g(x) = f(x+1) + 3f'(3x+10) \xrightarrow{x=-2} g'(-2) = f'(-1) + 3f'(4)$$

$$\xrightarrow{f'(-1) = f'(4)} g'(-2) = 4f'(-1) = 4 \times \frac{3}{2} = 6$$