

مریم یوسفی (سوال 1) ۲۰

۲۲ عدد

$$f(x) = \begin{cases} a + \sqrt{x} & ; x < 1 \\ b \sqrt{x} & ; x \geq 1 \end{cases}$$

اگر A همواره $a+b=5$

$a+|x| \Rightarrow x=0$

بر فرض $x=1 \rightarrow a=1$

$$\left\{ \begin{matrix} a+1 \\ b \end{matrix} \right\} \rightarrow a+1=b \Rightarrow a=\frac{1}{2}$$

$\Rightarrow a+b = \frac{1}{2} + \frac{1}{2} = 1$ ۲

فرض $x=1$

حاصل می شود

$$\frac{a}{\sqrt{x}} \rightarrow 1$$

$$\frac{2b}{2\sqrt{x}} \rightarrow \frac{2b}{2} \rightarrow \frac{2b}{2} = 1 \rightarrow b = \frac{1}{2}$$

۲

(سوال ۲)

$$f(x) = \frac{|x-1|}{\sqrt{x}} \quad g(x) = \frac{\sqrt{x^2-1} + x}{\sqrt{x}}$$

$f'(1) - 2g'(1) = ? \rightarrow (f-2g)'(1)$

$x=1 \rightarrow \frac{-(x-1) + \sqrt{x}}{\sqrt{x}}$

$$\rightarrow f(x) - 2g(x) = \frac{-(x-1)}{\sqrt{x}} + \frac{2(x-1) - 2\sqrt{x}}{\sqrt{x}} \Rightarrow \frac{-2\sqrt{x}}{\sqrt{x}}$$

$\Rightarrow -2\sqrt{x} x^{-\frac{1}{2}} \xrightarrow{\text{مشتق}} -2\sqrt{x} \left(-\frac{1}{2}\right) x^{-\frac{3}{2}} \Rightarrow x=1 \rightarrow -2\sqrt{x} \left(-\frac{1}{2}\right) (1)$

$= \left[\frac{\sqrt{x}}{1} \right]$ ۲

$$f(n) = \begin{cases} bn+c & n < a \\ \frac{1}{n} & n \geq a \end{cases} \quad ac = ? \quad (\text{سوال ۳})$$

! Case possible for both, $n=a$

$$n=a \begin{cases} ab+c \\ \frac{1}{a} \end{cases} \rightarrow ab+c = \frac{1}{a} \rightarrow -\frac{1}{a} + c = \frac{1}{a} \rightarrow c = \frac{2}{a}$$

$$f'(n) = \begin{cases} b & n < a \\ -\frac{1}{n^2} & n \geq a \end{cases} \rightarrow n=a \begin{cases} b \\ -\frac{1}{a^2} \end{cases} \rightarrow b = -\frac{1}{a^2} \quad (\text{۲})$$

$$ac = \frac{2}{a} \times a = \boxed{2}$$

$$f(n) = \begin{cases} b & n < a \\ b + (n-a)^m & n \geq a \end{cases} \quad (\text{سوال ۴})$$

$$\lim_{n \rightarrow a} f(n) \rightarrow b = b$$

$$\lim_{n \rightarrow a} f'(n) \rightarrow 0 \neq m(n-a)^{m-1} \rightarrow n=a \rightarrow 0 \neq 0 \times$$

$$\rightarrow m-1=0 \rightarrow m=1 \quad (\text{Case modified})$$

$$\rightarrow 1 \neq 0 \checkmark$$

$$f(n) = \frac{1}{\sqrt[n]{n-1}} \quad g(n) = \frac{1}{n^2 - \ln^2 n} \quad (\text{سوال ۵})$$

$$(fog)'(-\sqrt{e}) \quad \leftarrow \text{Differentiate}$$

$$\rightarrow f(n) = \frac{1}{\sqrt[n]{n}} \quad g(n) = \frac{1}{n^2} \rightarrow fog(n) = \frac{1}{\sqrt[n]{n^2}} \rightarrow fog(n) = n^{-\frac{2}{n}}$$

$$\rightarrow (fog)'(n) = \boxed{1}$$

آپادانا

$$y = -a - \omega \quad f'(x) - f(x) = 1$$

(4 سوال)

$$f(x) = -x a - \omega \quad f'(x) = -a \rightarrow f(x) = -0.1\omega, f'(x) = 1.1\omega$$

$$\rightarrow -a - (-x a - \omega) = 1 \Rightarrow -a + x a + \omega = 1 \Rightarrow x a = -1 \rightarrow a = -1.1\omega$$

$$\frac{f(x)}{f'(x)} = \frac{-0.1\omega}{1.1\omega} = \boxed{-\frac{1}{11}}$$

✓

(2)

$$f(x) = (x[x^r + \frac{1}{r}])^r + 1$$

$$g(x) = \frac{1}{\sqrt[r]{x^r - 1}}$$

(10 سوال)

$$(f \circ g)'(x) = g'(x) \times f'(g(x))$$

$$g'(x) \rightarrow (x^r - 1)^{-\frac{1}{r}} \rightarrow -\frac{1}{r} (x^r - 1)^{-\frac{1}{r} - 1} \times r x^{r-1}$$

$$\rightarrow x = \frac{r}{\sqrt{r}} \rightarrow -\frac{1}{r} \left(\frac{r}{\sqrt{r}} \right) \left(\frac{1}{r} \right)^{-\frac{1}{r} - 1} = -\frac{1}{\sqrt{r}}$$

$$g\left(\frac{r}{\sqrt{r}}\right) = \frac{1}{\sqrt[r]{\frac{1}{r}}} = \frac{r}{\sqrt{r}}$$

$$f'(x) \Rightarrow x \rightarrow r \rightarrow x^r \rightarrow \epsilon \rightarrow [x^r + \frac{1}{r}] = \epsilon$$

$$\rightarrow f(x) = (\epsilon x)^r \rightarrow 14x^r \rightarrow f'(x) = r \epsilon x \rightarrow f'(x) = 4\epsilon$$

$$(f \circ g)\left(\frac{r}{\sqrt{r}}\right) = \frac{-1/\sqrt{r} \times 4\epsilon}{-1/\sqrt{r}} = \boxed{4}$$

(10 سوال)

$$f'(-1) = \frac{r}{r} \quad g(x) = f(x+1) + f(r(x+1)) \quad g'(-r) = ?$$

$$g'(x) = f'(x+1) + r f'(r(x+1)) \quad x = -r \rightarrow g'(-r) = f'(-1) + r f'(\epsilon)$$

$$\omega \text{ solution} \rightarrow f'(-1) = f'(\epsilon) = \frac{r}{r} \rightarrow g'(-r) = \epsilon f'(-1) - \epsilon \times \frac{r}{r} = \boxed{4}$$

(2)

آپ اپنا

$$\lim_{h \rightarrow 0} \frac{f'(a+h) - f'(a-h) + f'(a)}{h(a-h)}$$

$$f(n) = (n-1)\sqrt{n+1} \quad (9 \text{ سوال})$$

$$f(a) = 2$$

جواب $\rightarrow \frac{-2f'(a)f(a) + f'(a)}{a}$

$$f'(n) = \sqrt{n+1} + \frac{(n-1)}{\sqrt{n+1}}$$

$$\rightarrow \frac{(-2 \times \frac{2a}{12} \times 2) + 2(\frac{2a}{12})}{a}$$

$$\rightarrow f'(a) = 2 + \frac{1}{12} = \frac{25}{12} \checkmark$$

$$= \frac{-\frac{2a}{12} + \frac{2a}{12}}{a} \rightarrow \frac{-\frac{a}{12} + \frac{a}{12}}{a} \quad (2)$$

$$\rightarrow \frac{-\frac{2a}{12} + \frac{2a}{12}}{a} \rightarrow \boxed{\frac{-a}{12}} \checkmark$$

$$y = x^2 - 5x + 2 \quad 4y - 3x = 1 \quad (10 \text{ سوال})$$

↓

$$\hookrightarrow \frac{dy}{dx} = \frac{3}{4} = \frac{1}{\frac{4}{3}}$$

$$y' = 2x - 5$$

$$\text{مع } \frac{1}{\frac{4}{3}} m' = -1 \rightarrow m' = -\frac{3}{4}$$

$$2x - 5 = -\frac{3}{4} \Rightarrow 2x = \frac{17}{4} \Rightarrow x = \frac{17}{8} \Rightarrow y = 2 \Rightarrow \boxed{(1, 2)} \quad (2)$$