"A" دوازدهم ریاضی "ریاضی" — مبینا کهالتی الحسینی

۱-

f پیوسته:

$$f(x)=\begin{cases} a+\sqrt{x^2} & x<1 \\ b\sqrt[3]{x^2} & x\geq 1\end{cases} \quad * \; a+1=b$$

($\sqrt{x^2}=|x|$)

$$f'(x)=\begin{cases} 1 & 0<x<1 \\ -1 & x<0 \\ \dfrac{2\times b}{3\sqrt[3]{x}} & x>1 \end{cases}$$

$\frac{2b}{3}=1$ یا $\frac{2b}{3}=-1$

$b=\frac{3}{2}$ یا $b=\frac{-3}{2}$

$a=\frac{1}{2}$ ، $a=\frac{-5}{2}$

$a+b=2$ ، $a+b=-4$

۲-

$$f(x)=\frac{|2x-4|}{\sqrt[3]{x^2}} \qquad g(x)=\frac{\sqrt{x^2-4x+4}+\sqrt{3x}}{\sqrt[3]{x^2}}$$

$f'(1)-2g'(1)=? \Rightarrow (f-2g)'(1)=?$

$$\frac{|2x-4|}{\sqrt[3]{x^2}}-\frac{2|x-2|+2\sqrt{3x}}{\sqrt[3]{x^2}}=\frac{-2\sqrt{3x}}{\sqrt[3]{x^2}}=\frac{-2\sqrt{3}x^{\frac{1}{2}}}{x^{\frac{2}{3}}}=-2\sqrt{3}x^{\frac{1}{2}-\frac{2}{3}}$$

$$=-2\sqrt{3}x^{\frac{-1}{6}} \Rightarrow (f-2g)'(x)=\frac{2\sqrt{3}}{6}x^{\frac{-7}{6}} \xrightarrow{x=1} \frac{2\sqrt{3}}{6}=\frac{\sqrt{3}}{3}$$

۳- f در $\mathbb{R}$ مشتق‌پذیر ← در $\mathbb{R}$ پیوسته

$$f(x)=\begin{cases} bx+c & x<a \\ \dfrac{1}{x} & x\geq a\end{cases}$$

$$ba+c=\frac{1}{a} \Rightarrow ba^2+ca=1 \; *$$

$ca=2$ ✓

$$f'(x)=\begin{cases} b & x<a \\ \dfrac{-1}{x^2} & x\geq a\end{cases}$$

$f'_+(a)=f'_-(a)$

$$\frac{-1}{a^2}=b \Rightarrow ba^2=-1 \; *$$

۴- m صحیح و نامنفی ؟

$$f(x)=\begin{cases} b & x<a \\ b+(x-a)^m & x\geq a \end{cases}$$

$L_2=\infty,\ L_1 \ldots\ L_1 \neq L_2 \rightarrow (a,b)$ گوشه‌ای

$$f'(x)=\begin{cases} 0 & x<a \quad f'_-(a)=0 \\ m(x-a)^{m-1} & x>a \quad f'_+(a)\neq 0 \end{cases}$$

If $m>1 \Rightarrow f'_+(a)=0$ ✗

If $m<1 \Rightarrow f'_+(a)=0$ ✗

If $m=1 \Rightarrow f'_+(a)=m$ ✓

✓ فقط به ازای یک مقدار صحیح و نامنفی $(m=1)$

۵-

$$f(x)=\frac{1}{\sqrt[3]{x-|x|}} \qquad g(x)=\frac{1}{x^3-|x^3|}$$

$$g'(-\sqrt[3]{2})\,f'(g(-\sqrt[3]{2}))=? \rightsquigarrow \left(f(g(-\sqrt[3]{2}))\right)'=?$$

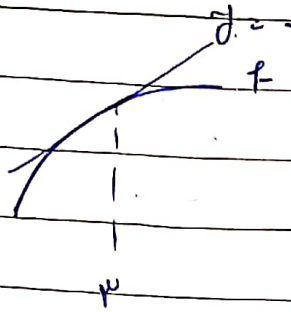
$$fog(x) \Rightarrow \frac{1}{\sqrt[3]{\frac{1}{x^3-|x^3|}-\left|\frac{1}{x^3-|x^3|}\right|}} = \frac{1}{\sqrt[3]{\frac{1}{2x^3}-\left|\frac{1}{2x^3}\right|}}$$

$x^3-|x^3|$ ... $-x^3$ ... $\frac{1}{2x^3}$

$$\frac{1}{\sqrt[3]{\frac{2}{2x^3}}}=\frac{1}{\frac{1}{x}}$$

$$\Rightarrow fog'(x)=1 \Rightarrow (fog)'(-\sqrt[3]{2})=1$$

-۶



$f'(۳) - f(۳) = ۲$ $\quad \frac{f(۳)}{f'(۳)} = ?$

$f'(۳) = -a$

$f(۳) = -۳a - b$ $\quad | \; -a + ۳a + b = ۲$

$۲a = -۲ \Rightarrow a = -۱ \Rightarrow f'(۳) = ۱$

$f(۳) = -۱$

$\Rightarrow \frac{f(۳)}{f'(۳)} = \frac{-۱}{۱}$

-۷

$f(x) = \left(x\left[x^۲ + \frac{۱}{۲}\right]\right)^۲ + ۱ \Rightarrow (f \circ g)'\left(\frac{۳}{\sqrt{۸}}\right)$

$g(x) = \frac{۱}{\sqrt[۳]{x^۲ - ۱}}$

$g'\left(\frac{۳}{\sqrt{۸}}\right) f'\left(g\left(\frac{۳}{\sqrt{۸}}\right)\right)$

$* g\left(\frac{۳}{\sqrt{۸}}\right) = \frac{۱}{\sqrt[۳]{\frac{۹}{۸} - ۱}} = ۲$ $\quad * f(x) = \left(x\left[x^۲ + \frac{۱}{۲}\right]\right)^۲ + ۱$

$* f(x) = ۱۶x^۴ + ۱ \Rightarrow f'(x) = ۶۴x \Rightarrow f'(۲) = ۶۴$

$g(x) = (x^۲ - ۱)^{\frac{-۱}{۳}} = \frac{-۱}{۳}(x^۲ - ۱)^{\frac{-۴}{۳}}(۲x) =$

$\frac{-۱}{۳} \times \frac{۱}{\sqrt[۳]{\left(\frac{۱}{۸}\right)^۴}} \times ۲ \times \frac{۳}{\sqrt{۸}} = \frac{-۱}{۳} \times ۱۶ \times \frac{۳}{۲\sqrt{۲}} = \frac{-۱۶}{\sqrt{۲}}$

$\frac{(f \circ g)'\left(\frac{۳}{\sqrt{۸}}\right)}{-۱۲۸\sqrt{۲}} = \frac{۶۴ \times ۲^۴ \times ۲}{۶۴ \times -۱۶} = ۴$

۸- f پیوسته → $T_f = 8 \Rightarrow f'(-1) = f'(\varepsilon) = \frac{3}{2}$

$f'(-1) = \frac{3}{2} \qquad g(x) = f(x+1) + f(3x+10) \Rightarrow g'(-2) = ?$

$g'(x) = f'(x+1) + 3f'(3x+10)$

$g'(-2) = f'(-1) + 3f'(-4) = \frac{3}{2} + \frac{9}{2} = \frac{12}{2} = 6$

$\Rightarrow \boxed{g'(-2) = 6}$

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۹ $f(x) = (x-4)\sqrt[3]{x+3}$

$\lim_{h\to 0} \frac{f^2(5-h) - 3f(5-h) + 2}{h(5-h) = 5h - h^2} = \frac{0}{0}$ hop →

$\lim_{h\to 0} \frac{-2f(5-h)f'(5-h) + 3f'(5-h)}{5-2h}$

$\Rightarrow \frac{-2f(5)f'(5) + 3f'(5)}{5}$

$f'(x) = 1\sqrt[3]{x+3} + \frac{1}{3\sqrt[3]{(x+3)^2}}(x-4)$

$* f'(5) = 2 + \frac{1}{12} \times 1 = \frac{25}{12}$

$* f(5) = 2$

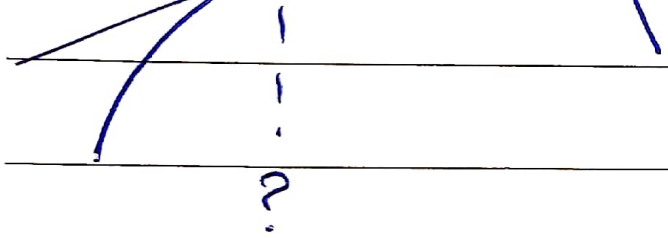
$\frac{-4 \times \frac{25}{12} + \frac{75}{12}}{5} = \frac{\cancel{-100+75}^{\,-25}}{12 \times 5} = \boxed{\frac{-5}{12}}$

$f(x)$

$y = x^2 - 4x + 5$

$4y - 3x = 1$

$f'(x) = 2x - 4 = -1 \Rightarrow 2x = 3 \Rightarrow \boxed{x = 1.5}$



$4y - 3x = 1$

$y = \frac{1 + 3x}{4} \rightarrow m = \frac{1}{2} \Rightarrow$ ميل العمودي $= -2$

$\Rightarrow$ تقطع $(1.5, 2)$ ✓