۱۶,۲۵

DATE / / SUBJECT: (سارا اسینی زاد)

① $f(x)=\begin{cases} a+\sqrt{x^2} & x<1 \\ b\sqrt[3]{x^2} & x\geq 1\end{cases} \xrightarrow{x=1} a+1=b$

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مشتق سمت چپ $f(x)=a+\sqrt{x^2}=a+|x| \Rightarrow f'(x)=1 \quad f'_-(1)=1$

مشتق سمت راست $b x^{\frac{2}{3}} \Rightarrow f'(x)=\frac{2}{3}bx^{-\frac{1}{3}} \to f'_+(1)=\frac{2b}{3} \quad 1\neq \frac{2b}{3} \quad b\neq \frac{3}{2}$

$a=b-1 \Rightarrow a+b=b-1+b=2b-1 \quad a+b=2b-1=2$

② $f(x)=\frac{2(2-x)}{x^{\frac{2}{3}}} \quad f'(x)=\frac{-2x^{\frac{2}{3}}-\frac{4}{3}(2-x)x^{-\frac{1}{3}}}{x^{\frac{4}{3}}} \Rightarrow f'(1)=-2$

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$g(x)=\frac{(2-x)+\sqrt{6x}}{x^{\frac{2}{3}}} \to g'(1)=\frac{-1+\frac{3}{2\sqrt{3}}\times 1-(1+\sqrt{6})\cdot\frac{2}{3}}{1} \quad g'(1)=-\frac{1}{2} \quad -2-2(-\frac{1}{2})=-1$

(جواب پایین صفحه)

③ $ab+c=\frac{1}{a} \quad f'(x)\begin{cases} b & x<a\\ -\frac{1}{x^2} & x>a\end{cases} \Rightarrow f'_-(a)=b, \quad f'_+(a)=-\frac{1}{a^2} \to b=-\frac{1}{a^2}$

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$a(-\frac{1}{a^2})+c=\frac{1}{a} \to c=\frac{2}{a} \to \boxed{ac=2}$ ✓

④ $x=a \to f(x)$ پیوسته ✓ $\quad f'(x)\begin{cases}0 & x<a\\ m(x-a)^{m-1} & x>a\end{cases} \quad f'_-(a)=0 \to f'_+(a)\neq 0$

$m(a-a)^{m-1}\neq 0 \to m=1$ ✓

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⑤ $(f\circ g)'(-\sqrt[4]{2}) \quad D_g(-\infty,0) \quad g(x)=\frac{1}{2x^4} \xrightarrow{a<0} f(g(x))=\frac{1}{\sqrt[4]{\frac{1}{2x^4}+\frac{1}{2x^4}}}=\sqrt[4]{x}=x$

$(f\circ g)'(x)=1$ ✓

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⑥ $-a-(-3a-a)=2 \to -a+3a+a=2 \to 2a=-3 \to a=-\frac{3}{2}$

$f(3)=\frac{3}{2}, \quad f(c)=-\frac{1}{2} \quad \frac{f(c)}{f'(c)}=-\frac{1}{3}$ ✓

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$y=f(g(x)) \rightarrow y'=g'(x)f'(g(x)) \xrightarrow{x=\frac{3}{\sqrt{8}}} y'=g'(\frac{3}{\sqrt{8}})f'(\underbrace{g(\frac{3}{\sqrt{8}})}_{2}) \rightarrow y'=g'(\frac{3}{\sqrt{8}})f'(2)$

$g(x)=\frac{1}{\sqrt[3]{x^2-1}}=(x^2-1)^{-\frac{1}{3}} \rightarrow g'(x)=-\frac{1}{3}(x^2-1)^{-\frac{4}{3}}\times 2x=-\frac{2}{3}x(x^2-1)^{-\frac{4}{3}}$

(سوال امتحانی)

(7) $g'(\sqrt[5]{3})\times f'(g(\sqrt[5]{3}))=(f\circ g)'(\sqrt[5]{3})$ $g'(\frac{3}{\sqrt{8}})=\frac{-2}{3}\times\frac{3}{\sqrt{8}}(\frac{1}{8})^{-\frac{4}{3}}=\frac{-2}{\sqrt{8}}\times 2^4=-\frac{2^5\times\sqrt{2}}{2^2}=-2^3\sqrt{2} \rightarrow g'(\frac{3}{\sqrt{8}})=-8\sqrt{2}$

$f(x)=16x^2+1 \rightarrow f'(x)=32x \rightarrow f'(2)=64$

$y'=g'(\frac{3}{\sqrt{8}})f'(2)=-8\sqrt{2}\times 64=4\times(-128\sqrt{2}) \rightarrow$ ۴ برابر

(8) $g(x)=f(x+1)+f(3x+10) \rightarrow g'(x)=f'(x+1)+3f'(3x+10)$

$g'(-2)=f'(-1)+3f'(4) \rightarrow f(x+5)=f(x) \Rightarrow f'(x+5)=f'(x) \rightarrow f'(4)=f'(-1)$

$g'(-2)=\frac{3}{2}+3(\frac{3}{2})=6$

(2)

(9) $f(x)=(x-4)\sqrt[3]{x+5} \quad , \quad \sqrt[3]{x+3}+(x-4)\times\frac{1}{3(\sqrt[3]{x+3})^2}$

$\lim\frac{f^2(5-h)-3f(5-h)+2}{h(5-h)}=\lim\frac{f(5-h)-1}{5-h}\times-\frac{f(5-h)-2}{-h}=\frac{f(5)-1}{5}(-f'(5))$

(2)

$\frac{1}{5}\times-\frac{25}{12}=\frac{-5}{12}$

(10) $4y-3x=1 \quad m=\frac{+3}{4}=\frac{1}{4}$ $\quad$ شیب عمود $=-2$ $\quad y'=2x-4=-2 \rightarrow x=1 \quad y=2$

قرینه معکوس $\quad (1,2)$

(2)

سوال ۱۲

$f(x)=\frac{|2x-4|}{\sqrt[3]{x^2}} \xrightarrow{x=1} \frac{-2x+4}{\sqrt[3]{x^2}}$

$f'(x)=\left(\frac{-2x+4}{\sqrt[3]{x^2}}\right)'=\frac{-2(1)-\frac{2}{3\sqrt[3]{x}}(2)}{\sqrt[3]{x^4}} \rightarrow f'(1)=\frac{-2(1)-\frac{2}{3}(2)}{1}=\frac{-10}{3}$

$g(x)=\frac{\sqrt{x^2-4x+4}+\sqrt{3x}}{\sqrt[3]{x^2}}=\frac{|x-2|+\sqrt{3x}}{\sqrt[3]{x^2}} \xrightarrow{x=1} \frac{(-x+2)+\sqrt{3x}}{\sqrt[3]{x^2}}$

$g'(x)=\frac{(-1+\frac{3}{2\sqrt{3x}})(1)-\frac{2}{3\sqrt[3]{x}}(1+\sqrt{3})}{\sqrt[3]{x^4}} \rightarrow g'(1)=\frac{(-1+\frac{3}{2\sqrt{3}})-\frac{2}{3}(1+\sqrt{3})}{1}=-1+\frac{3}{2\sqrt{3}}-\frac{2}{3}-\frac{2\sqrt{3}}{3}=-\frac{5}{3}-\frac{\sqrt{3}}{6}$

$f'(1)-2g'(1)=\frac{-10}{3}-2(\frac{-5}{3}-\frac{\sqrt{3}}{6})=\frac{\sqrt{3}}{3}$