حل تمرین

دوازدهم دفتر A.

پرنیان شمشیری - تکلیف شماره ۲۲

سوال ۱

$$f(x) = \begin{cases} a+\sqrt{x^2} & x<1 \\ b\sqrt[3]{x^2} & x\geq 1 \end{cases} = a+|x|$$

طبق ضابطه نتیجه می‌گیریم که تابع در نقطه $x=0$ مشتق ندارد زیرا: پس تابع در نقاط پیوسته در $x=0$ مشتق ندارد.

$\rightarrow$ تابع پیوسته $\Rightarrow a+\sqrt{x^2} = b\sqrt[3]{x^2} \rightarrow \boxed{a+1=b}$

$f'_+(1) = a+|x| = a+x = 1$

$f'_-(1) = bx^{\frac{2}{3}} = \frac{2}{3}bx^{-\frac{1}{3}} \xrightarrow{x=1} \frac{2}{3}b$

$\frac{2}{3}b = 1 \rightarrow b=\frac{3}{2}$

$b=\frac{3}{2}$ و $a=\frac{1}{2} \rightarrow a+b=2$

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سوال ۲

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$$f(x) = \frac{|2x-4|}{\sqrt[3]{x^2}} \xrightarrow{x=1} f(x) = \frac{4-2x}{\sqrt[3]{x^2}} \longrightarrow f'(x) = \frac{(-2)(\sqrt[3]{x^2}) - (\frac{2}{3}x^{-\frac{1}{3}})(4-2x)}{(\sqrt[3]{x^2})^2}$$

$$\rightarrow f'(1) = \frac{((-2)(1)) - ((\frac{2}{3})(2))}{1} = -2 - \frac{4}{3} = \frac{-10}{3}$$

$$g(x) = \frac{\sqrt{x^2-4x+4}+\sqrt{3x}}{\sqrt[3]{x^2}} = \frac{|x-2|+\sqrt{3x}}{\sqrt[3]{x^2}}$$

$$g'(x) = \frac{(-1+\frac{3}{2\sqrt{3x}})(1) - \frac{2}{3\sqrt[3]{x}}(1+\sqrt{3})}{\sqrt[3]{x^4}} \rightarrow g'(1) = \frac{(-1+\frac{3}{2\sqrt{3}}) - \frac{2}{3}(1+\sqrt{3})}{1} = -\frac{5}{3} - \frac{\sqrt{3}}{6}$$

$$g(x) = \frac{\sqrt{x^2-4x+4}+\sqrt{3x}}{\sqrt[3]{x^2}} = \frac{\sqrt{(x-2)^2}+\sqrt{3x}}{\sqrt[3]{x^2}} = \frac{|x-2|+\sqrt{3x}}{\sqrt[3]{x^2}} = \frac{2-x+\sqrt{3x}}{\sqrt[3]{x^2}} \rightarrow g'(x) = \frac{((1+\frac{3}{2\sqrt{3x}})(\sqrt[3]{x^2})) - ((\frac{2}{3}x^{-\frac{1}{3}})(2-x+\sqrt{3x}))}{(\sqrt[3]{x^2})^2}$$

$$\longrightarrow g'(1) = \frac{((1+\frac{3}{2\sqrt{3}})(1)) - ((\frac{2}{3})(1+\sqrt{3}))}{1} = \frac{(1+\frac{\sqrt{3}}{2}) - (\frac{2}{3}+\frac{2\sqrt{3}}{3})}{1} = \frac{2}{2}+\frac{\sqrt{3}}{2}-\frac{2}{3}-\frac{2\sqrt{3}}{3} = \frac{6+3\sqrt{3}-4-4\sqrt{3}}{6} = \frac{2-\sqrt{3}}{6}$$

$$\rightarrow f'(1) - 2g'(1) = (\frac{-10}{3}) - 2(\frac{2-\sqrt{3}}{6}) = \frac{-10}{3} \cdot \frac{-2+\sqrt{3}}{3} = \boxed{\frac{-12+\sqrt{3}}{3}}$$

$$f'(1) - 2g'(1) = \frac{-10}{3} - 2(\frac{-5}{3} - \frac{\sqrt{3}}{6}) = \frac{\sqrt{3}}{3}$$

سوال ۳

$$f(x) \begin{cases} bx+c & x<a \\ \frac{1}{x} & x\geq a \end{cases}$$

مشتق پذیر $\rightarrow$ پیوسته است $\rightarrow ab+c = \frac{1}{a} \xrightarrow{\times a} a^2b+ac=1$

مشتق برابر $\rightarrow b = \frac{-1}{a^2} \rightarrow ba^2=-1$

$\boxed{ac=2}$

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سوال ۴

$$\left.\begin{array}{l} f'_+(a) = 0 \\ f'_-(a) = m(x-a)^{m-1} \end{array}\right\} m(x-a)^{m-1} \neq 0 \xrightarrow{m>0} \boxed{m=1} \rightarrow$$

یک مقدار برای m

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s.a.m

$$fog(x) = f\left(\frac{1}{2x^3}\right) = \frac{1}{\sqrt[3]{2\times\left(\frac{1}{2x^3}\right)}} = x \to fog(x) = x \to (fog(x))' = 1$$

سوال ۵ (۱)

$$g(x) = \frac{1}{x^3 - |x^3|} \xrightarrow{x = -\sqrt[3]{2}} g(x) = \frac{1}{2x^3} = \frac{1}{2}x^{-3} \to g'(x) = \left(\frac{1}{2}\right)(-3)x^{-4} = \frac{-3}{2}x^{-4}$$

$$f(x) = \frac{1}{\sqrt[3]{x - |x|}} \xrightarrow{g(-\sqrt[3]{2})} f(x) = \frac{1}{\sqrt[3]{2x}} = \frac{1}{\sqrt[3]{2}} \times \frac{1}{\sqrt[3]{x}} = \frac{1}{\sqrt[3]{2}} \times x^{\frac{-1}{3}} \to \boxed{f'(x) = \frac{1}{\sqrt[3]{2}} \times \left(\frac{-1}{3}\right) x^{\frac{-4}{3}}}$$

$$g'(-\sqrt[3]{2}) = \frac{-3}{2}(-\sqrt[3]{2})^{-4} =$$

سوال ۶ (۲)

$$y = -ax - 5 \xrightarrow{f(3)} f(3) = -3a - 5 \quad , \quad f'(x) = -a$$

$$\to f'(3) - f(3) = 2 \to -a - (-3a - 5) = 2 \to -a + 3a + 5 = 2 \to 2a + 5 = 2 \to 2a = -3$$

$$\to a = \frac{-3}{2} \to y = \frac{3}{2}x - 5 \to f(3) = \frac{9}{2} - 5 = \frac{-1}{2}, \quad f'(3) = \frac{3}{2} \quad \Big| \quad \frac{f(3)}{f'(3)} = \frac{\frac{-1}{2}}{\frac{3}{2}} = \frac{-1}{3}$$

سوال ۷ (۰)

$$y = f(g(x)) \to y' = g'(x) f'(g(x)) \xrightarrow{x = \frac{3}{\sqrt{8}}} y' = g'\left(\frac{3}{\sqrt{8}}\right) f'\left(\underbrace{g\left(\frac{3}{\sqrt{8}}\right)}_{2}\right) \to y' = g'\left(\frac{3}{\sqrt{8}}\right) f'(2)$$

$$g(x) = \frac{1}{\sqrt[3]{x^2 - 1}} = (x^2 - 1)^{-\frac{1}{3}} \to g'(x) = -\frac{1}{3}(x^2 - 1)^{-\frac{4}{3}} \times 2x = -\frac{2}{3}x(x^2 - 1)^{-\frac{4}{3}}$$

۴ برابر

$$g'\left(\frac{3}{\sqrt{8}}\right) = \frac{-2}{3} \times \frac{3}{\sqrt{8}}\left(\frac{1}{8}\right)^{-\frac{4}{3}} = \frac{-2}{\sqrt{8}} \times 2^4 = -\frac{2^4 \times \sqrt{2}}{2^2} = -4\sqrt{2} \to g'\left(\frac{3}{\sqrt{8}}\right) = -8\sqrt{2}$$

$$y' = g'\left(\frac{3}{\sqrt{8}}\right) f'(2) = -8\sqrt{2} \times 64 = 4 \times (-128\sqrt{2})$$

$$f(x) = 16x^2 + 1 \to f'(x) = 32x \to f'(2) = 64$$

سوال ۸ (۲)

$$g(x) = f(x+1) + f(3x+10) \to g'(x) = f'(x+1) + 3f'(3x+10)$$

$$\xrightarrow{x = -2} g'(-2) = f'(-1) + 3f'(4) \xrightarrow[f'(-1) = f'(4)]{\text{تابع با شیب ثابت داده است}} g'(-2) = 4f'(-1) = 4 \times \frac{3}{2} = \boxed{6}$$

سوال ۹ (۱) باطل

$$\lim_{h\to 0} \frac{f^2(5-h) - 3f(5-h) + 2}{h(5-h)} \quad , \quad f(x) = (x-4)\sqrt[3]{x+3} \to f(5) = 2$$

$$\lim_{h\to 0} \frac{(f(5-h) - 2)(f(5-h) - 1)}{h(5-h)} \to \frac{\overbrace{(f(5-h) - f(5))}^{f'(5)}(f(5-h) - 1)}{h(5-h)} = \frac{f'(5)(1)}{5} = \frac{f'(5)}{5}$$

$$-\frac{5}{12} \qquad \frac{\frac{25}{12}}{5} = \cancel{\frac{5}{12}}$$

$$f'(x) = (1)(\sqrt[3]{x+3}) + (x-4)\left(\frac{1}{3\sqrt[3]{(x+3)^2}}\right) \Rightarrow f'(5) = (1)(2) + (1)\left(\frac{1}{12}\right) = \frac{25}{12}$$

سوال ۱۰ (۲)

$$y = x^2 - 4x + 5 \to y' = 2x - 4$$

$$6y - 3x = 1 \to y = \frac{3}{9}x + 1 \to m = \frac{3}{9} \quad , \quad mm' = -1 \to m' = \frac{-9}{3}$$

$$2x - 4 = -2 \to x = 1 \to y = 2$$

$$\to 1\tfrac{1}{2}$$

s.a.m