

$$f(x) = \begin{cases} a + \sqrt{x^3} & x < 1 \\ b \sqrt[3]{x^3} & x \geq 1 \end{cases} = a + |x|$$

سوال ۱

طبق ضابطه تغییر می گیریم که تابع در نقطه $x=0$ مشتق ندارد زیرا:  پس تابع در همه نقاط پیوسته و در $x=0$ مشتق ندارد.

$$\rightarrow \text{تابع پیوسته} \Rightarrow a + \sqrt{x^3} = b \sqrt[3]{x^3} \rightarrow \boxed{a+1=b} \quad \left\{ \begin{array}{l} b = \frac{3}{4}, a = \frac{1}{4} \rightarrow a+b = 1 \end{array} \right.$$

$$\begin{cases} f'_+(1) = a + |x| = a + x = 1 \\ f'_-(1) = b x^{\frac{3}{4}} = \frac{3}{4} b x^{-\frac{1}{4}} \xrightarrow{x=1} \left(\frac{3}{4} b \right) \end{cases} \quad \left\{ \begin{array}{l} \frac{3}{4} b = 1 \rightarrow b = \frac{4}{3} \end{array} \right.$$

$$f(x) = \frac{|x-1|}{\sqrt[3]{x^3}} \xrightarrow{x=1} f(x) = \frac{x-1}{\sqrt[3]{x^3}} \rightarrow f'(x) = \frac{(1)(\sqrt[3]{x^3}) - (x-1)(\frac{1}{3}x^{-\frac{2}{3}})}{(\sqrt[3]{x^3})^2}$$

سوال ۲

$$\rightarrow f'(1) = \frac{((-1)(1)) - \left(\left(\frac{4}{3}\right)(1)\right)}{1} = -1 - \frac{4}{3} = \left(\frac{-7}{3}\right)$$

$$g(x) = \frac{\sqrt{x^3-4x+4} + \sqrt{x^3}}{\sqrt[3]{x^3}} = \frac{\sqrt{(x-2)^2 + \sqrt{x^3}}}{\sqrt[3]{x^3}} = \frac{|x-2| + \sqrt{x^3}}{\sqrt[3]{x^3}} \rightarrow g'(x) = \frac{\left(1 + \frac{3}{4\sqrt{x^3}}\right)\left(\frac{3}{4}\sqrt{x^3}\right) - \left(\frac{3}{4}\sqrt{x^3}\right)\left(\frac{1}{3}\sqrt{x^3}\right)}{\left(\sqrt[3]{x^3}\right)^2}$$

$$\rightarrow g'(1) = \frac{\left(\left(1 + \frac{3}{4\sqrt{1}}\right)(1)\right) - \left(\left(\frac{3}{4}\right)(1 + \sqrt{1})\right)}{1} = \left(1 + \frac{\sqrt{3}}{4}\right) - \left(\frac{3}{4} + \frac{3\sqrt{3}}{4}\right) = \frac{1}{4} + \frac{\sqrt{3}}{4} - \frac{3}{4} - \frac{3\sqrt{3}}{4} = \frac{-2 - 2\sqrt{3}}{4} = \left(\frac{-1 - \sqrt{3}}{2}\right)$$

$$\rightarrow f'(1) - 2g'(1) = \left(\frac{-7}{3}\right) - 2\left(\frac{-1 - \sqrt{3}}{2}\right) = \frac{-7}{3} + \frac{1 + \sqrt{3}}{1} = \frac{-12 + \sqrt{3}}{3}$$

$$f(x) = \begin{cases} bx+c & x < a \\ \frac{1}{x} & x \geq a \end{cases} \quad \begin{array}{l} \text{پیوسته است} \rightarrow ab+c = \frac{1}{a} \xrightarrow{\times a} \underline{ab+ac=1} \\ \text{مشتق یزر} \rightarrow \text{مشتق برابر} \rightarrow b = \frac{-1}{a^2} \rightarrow \underline{ba^2=-1} \end{array} \quad \boxed{ac=2}$$

سوال ۳

$$\begin{cases} f'(a) = 0 \\ f'(a) = m(a-a)^{m-1} \end{cases} \quad \left\{ \begin{array}{l} m(a-a)^{m-1} \neq 0 \xrightarrow{m>0} \boxed{m=1} \end{array} \right.$$

سوال ۴

$$g(x) = \frac{1}{x^3 - 1x^3} \xrightarrow{x = -\sqrt[3]{y}} g(x) = \frac{1}{x^3 - 1x^3} = \frac{1}{x^3} \rightarrow g'(x) = \left(\frac{1}{x^3}\right)' = \left(-\frac{1}{x^4}\right) = \left(-\frac{1}{y^4}\right)$$

سوال (5)

$$f(x) = \frac{1}{\sqrt[3]{x-1x}} \xrightarrow{g(-\sqrt[3]{y})} f(x) = \frac{1}{\sqrt[3]{x-1x}} = \frac{1}{\sqrt[3]{x}} \times \frac{1}{\sqrt[3]{x}} = \frac{1}{\sqrt[3]{x}} \times x^{-\frac{1}{3}} \rightarrow f'(x) = \frac{1}{\sqrt[3]{x}} \times \left(-\frac{1}{x^{\frac{4}{3}}}\right) = \left(-\frac{1}{x^{\frac{5}{3}}}\right)$$

$$g'(-\sqrt[3]{y}) = \frac{1}{y^{\frac{5}{3}}} (-\sqrt[3]{y})^{-\frac{4}{3}} = \dots$$

$$y = -ax - \Delta \xrightarrow{f(x)} f(x) = -x^3 a - \Delta, f'(x) = -3a$$

سوال (6)

$$\rightarrow f'(x) - f'(x) = y \rightarrow -a - (-3a - \Delta) = y \rightarrow -a + 3a + \Delta = y \rightarrow 2a + \Delta = y \rightarrow 2a = y - \Delta$$

$$\rightarrow \left(\alpha = \frac{-y}{2}\right) \rightarrow y = \frac{y}{2} x - \Delta \rightarrow f(x) = \frac{y}{2} x - \Delta = \left(\frac{-1}{2}\right) \left\{ \begin{array}{l} f(x) = \frac{-1}{2} \\ f'(x) = \frac{y}{2} \end{array} \right. \rightarrow \frac{f(x)}{f'(x)} = \frac{-1}{\frac{y}{2}} = \left(\frac{-2}{y}\right)$$

سوال (7)

$$g(x) = f(x+1) + f(x^3+10) \rightarrow g'(x) = f'(x+1) + 3x^2 f'(x^3+10)$$

سوال (8)

$$\xrightarrow{x \rightarrow -1} g'(-1) = f'(-1) + 3x^2 f'(x) \xrightarrow{\text{بموجب باء باء باء}} g'(-1) = f'(-1) = f' \times \frac{1}{y} = \boxed{4}$$

$$\lim_{h \rightarrow 0} \frac{f(\Delta-h) - 3f(\Delta-h) + 2}{h(\Delta-h)}, f(x) = (x-4)\sqrt{x+3} \rightarrow f(\Delta) = 2$$

سوال (9)

$$\lim_{h \rightarrow 0} \frac{(f(\Delta-h) - 2)(f(\Delta-h) - 1)}{h(\Delta-h)} \rightarrow \frac{(f(\Delta-h) - f(\Delta))(f(\Delta-h) - 1)}{h(\Delta-h)} = \frac{f'(\Delta)(1)}{\Delta} = \frac{f'(\Delta)}{\Delta} \left\{ \begin{array}{l} \frac{25}{12} = \frac{5}{12} \end{array} \right.$$

$$f'(x) = (1)(\sqrt{x+3}) + (x-4)\left(\frac{1}{2\sqrt{x+3}}\right) \Rightarrow f'(\Delta) = (1)(2) + (1)\left(\frac{1}{2}\right) = \frac{5}{2}$$

سوال (10)

$$y = x^2 - 4x + \Delta \rightarrow y' = 2x - 4$$

$$4y - 2x = 1 \rightarrow y = \frac{2}{4}x + 1 \rightarrow m = \frac{2}{4}, mm' = -1 \rightarrow m' = \frac{-4}{2}$$

$$\left\{ \begin{array}{l} 2x - 4 = -2 \rightarrow x = 1 \rightarrow y = 2 \\ \rightarrow \left(\frac{1}{2}\right) \end{array} \right.$$

s.a.m