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تکالیف شماره ۲۲

هانیه کوشکیان

روز دهم دفتر A

$$f(x) = \begin{cases} a + \sqrt{x^p} & x < 1 \\ b\sqrt{x^p} & x > 1 \end{cases}$$

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$$\sqrt{x^p} = |x|, \quad \sqrt[p]{x^p} = (x^p)^{\frac{1}{p}} = |x|^{\frac{p}{p}} = |x|$$

نقطه پیوستگی در $x=1$ است

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$$

$$\left. \begin{aligned} \text{بسمت چپ} &= a + 1 = a + 1 \\ \text{بسمت راست} &= b \cdot 1 = b \end{aligned} \right\} \Rightarrow b = a + 1$$

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تابع پیوسته در $x=1$ است

$$\frac{d}{dx} (a + |x|) = 1 \quad (x > 0)$$

مشتق از چپ

$$\frac{d}{dx} (bx^{\frac{1}{p}}) = \frac{1}{p} b x^{-\frac{1}{p}} \Rightarrow \frac{1}{p} b$$

مشتق از راست

$$\Rightarrow 1 = \frac{1}{p} b \Rightarrow b = \frac{1}{p} \Rightarrow a = b - 1 = \frac{1}{p} - 1$$

$$\Rightarrow a + b = 1$$

$$f(x) = \frac{1}{\sqrt{x}} - 1$$

$$g(x) = \frac{\sqrt{x^2 - 1} + x + \sqrt{x}}{\sqrt{x^2}}$$

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$$f'(1) = g'(1) \Rightarrow f'(1) \Rightarrow \frac{1}{2} - 1 = -\frac{1}{2} < 0 \Rightarrow 1 - 1 = 0$$

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$$\Rightarrow f(x) = \frac{1}{\sqrt{x}} \Rightarrow \frac{1}{\sqrt{x}} - 1 \Rightarrow \frac{1}{\sqrt{x}} - 1 - (-1) = \frac{1}{\sqrt{x}} - 1 + 1 = \frac{1}{\sqrt{x}}$$

$$g(x) = \frac{\sqrt{x^2 - 1} + x + \sqrt{x}}{\sqrt{x^2}} = \frac{|x-1| + \sqrt{x^2 - 1}}{\sqrt{x^2}} \xrightarrow{x=1} \frac{(-1+1) + \sqrt{1}}{\sqrt{1}} = 1$$

Kian

$$g'(x) = \frac{(-1 + \frac{1}{\sqrt{x}})'(1) - \frac{1}{\sqrt{x}}(1 + \sqrt{x})}{\sqrt{x^2}} \rightarrow g'(1) = \frac{(-1 + \frac{1}{\sqrt{1}})' - \frac{1}{\sqrt{1}}(1 + \sqrt{1})}{1} = -\frac{1}{2} - \frac{\sqrt{1}}{2}$$

$$f'(1) - g'(1) = \frac{1}{2} - (-\frac{1}{2} - \frac{\sqrt{1}}{2}) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

$$g(x) = \frac{\sqrt{x^2 - 1} + x}{x^{\frac{r}{r}}}$$

$$g'(1) = \frac{d(-1 - \sqrt{2})}{dx} - \frac{d(-1 - \sqrt{2})}{dx} \Rightarrow f'(1) - g'(1) = \frac{d(-1 - \sqrt{2})}{dx} \cdot \frac{1}{\sqrt{1+2}}$$

$$f(x) = \frac{1}{\sqrt{1-x}}$$

$$g(x) = \frac{1}{x^r \ln x}$$

$$g(-\sqrt{2}) f(g(-\sqrt{2}))$$

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حل:

$$m < 0 \Rightarrow |x|^m = -x^m \Rightarrow m^r - \ln m^r = m^r \Rightarrow g(m) = \frac{1}{m^r} = (m^r)^{-1} \rightarrow g'(m) = -\frac{r}{m^{r+1}}$$

$$\log m = f\left(\frac{1}{m^r}\right) = \frac{1}{\sqrt{1 - \frac{1}{m^r}}} = x \rightarrow f(g(m)) = x \rightarrow (f(g(m)))' = 1$$

سوال ۱۱ تابع $|x|$ در $x=0$ نقطه گوشه دارد پس مشتق ناپذیر است.

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) \rightarrow a+b = -1 \rightarrow a-b = -1 \rightarrow a-b = -1 \rightarrow a - \frac{r}{r} = -1 \rightarrow a = \frac{1}{r}$$

$$f'_+(1) = f'_-(1) \rightarrow 1 = \frac{r}{r} b x a^{-\frac{1}{r}} \rightarrow 1 = \frac{r}{r} b (1)^{-\frac{1}{r}} \rightarrow b = \frac{r}{r}$$

$$a+b = \frac{r}{r} + \frac{1}{r} = \frac{r}{r} = r$$

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) \rightarrow \frac{1}{a} = ba + c$$

$$f'(a) = \begin{cases} b & x < a \\ \frac{1}{a^r} & x > a \end{cases} \rightarrow f'_-(a) = f'_+(a) \rightarrow b = \frac{1}{a^r}$$

$$\frac{r}{a} = c \quad ac = r$$

سوال ۱۳

$$f'_-(a) \neq f'_+(a) \rightarrow \text{نقطه گوشه}$$

$$m=0 \rightarrow f'_-(a) = f'_+(a) \times \infty$$

$$m=1 \rightarrow \begin{cases} f'_-(a) = 0 \\ f'_+(a) = 1 \end{cases} \rightarrow f'_-(a) \neq f'_+(a)$$

$$m > r \rightarrow \begin{cases} f'_-(a) = 0 \\ f'_+(a) = 0 \end{cases} \rightarrow f'_-(a) = f'_+(a)$$

یعنی مقدار صحیح m

سوال ۱۴

سوال 4

$$f(u) = -3a - \omega, \quad f'(u) = -a$$

$$f'(u) - f(u) = 2 \rightarrow (-a) - (-3a - \omega) = 2 \rightarrow a = \frac{-\omega}{2}$$

$$\frac{f(u)}{f'(u)} = \frac{-3a - \omega}{-a} = \frac{-3(-\frac{\omega}{2}) - \omega}{\frac{\omega}{2}} = \frac{-1}{2}$$

$$y = f(g(u)) \rightarrow y' = g'(u) f'(g(u)) \xrightarrow{u = \frac{v}{\sqrt{\lambda}}} y' = g'(\frac{v}{\sqrt{\lambda}}) f'(g(\frac{v}{\sqrt{\lambda}})) \rightarrow y' = g'(\frac{v}{\sqrt{\lambda}}) f'(r) \quad \text{سوال 4}$$

$$g(u) = \frac{1}{\sqrt{2u-1}} = (2u-1)^{-\frac{1}{2}} \rightarrow g'(u) = -\frac{1}{2} (2u-1)^{-\frac{3}{2}} \times 2 = -\frac{1}{2} (2u-1)^{-\frac{3}{2}}$$

$$g'(\frac{v}{\sqrt{\lambda}}) = -\frac{1}{2} \times \frac{v}{\sqrt{\lambda}} \left(\frac{1}{\lambda}\right)^{-\frac{3}{2}} = -\frac{1}{\sqrt{\lambda}} \times v^{\frac{3}{2}} = -\frac{v^{\frac{3}{2}} \times \sqrt{2}}{v^{\frac{3}{2}}} = -v^{\frac{3}{2}} \sqrt{2} \rightarrow \underline{g'(\frac{v}{\sqrt{\lambda}}) = -\sqrt{2}v}$$

$$f(u) = 4u^2 + 1 \rightarrow f'(u) = 8u \rightarrow \underline{f'(r) = 4r}$$

$$y' = g'(\frac{v}{\sqrt{\lambda}}) f'(r) = -\sqrt{2}v \times 4r = r \times (-4\sqrt{2}v) \rightarrow \underline{r \times (-4\sqrt{2}v)}$$

$$g'(u) = f'(u+1) + 3f'(3u+1) \xrightarrow{u=-1} g'(-1) = f'(-1) + 3f'(1)$$

سوال 1

$$\underline{f'(-1) = f'(1)} \rightarrow g'(-1) = 4f'(-1) = 4 \times \frac{1}{4} = 1$$

$$f(u) = (u-1)^2 \sqrt{u+2} \rightarrow f'(u) = 2(u-1)\sqrt{u+2} + (u-1)^2 \frac{1}{2\sqrt{u+2}} \rightarrow f'(\omega) = 2 + \frac{1}{12} = \frac{25}{12}$$

سوال 9

$$\lim_{h \rightarrow 0} \frac{f'(\omega-h) - 3f'(\omega-h) + 2}{h(\omega-h)} = \lim_{h \rightarrow 0} \frac{-2f'(\omega-h)f(\omega-h) + 4f'(\omega-h)}{\omega-2h} =$$

$$\frac{-2f'(\omega)f(\omega) + 4f'(\omega)}{\omega} = \frac{-2 \times \frac{25}{12} \times 2 + 4 \times \frac{25}{12}}{\omega} = \underline{\frac{-\omega}{12}}$$

$$4y - 3u = 1 \rightarrow \text{شیب خط} = \frac{1}{4} \rightarrow \text{شیب خط عمود} = -4$$

سوال 10

$$y' = 3u - 4 = -4 \rightarrow u = 1, y = 2 \rightarrow \underline{(1, 2) \text{ نقطه}}$$