دوازدهم A

پیما_گام دوم

سوال ۱ـ

$$f(x)\begin{cases} a+\sqrt{x^2} & ; x<1 \\ b\sqrt[3]{x^2} & ; x\geq 1 \end{cases} \longrightarrow \text{پیوسته در } \mathbb{R}$$

باید در $x=1$ → در نقطه ناپیوسته است

$$x=1 \to a+1=b \qquad \lim \begin{cases} a+|x| \xrightarrow{x>0} a+1=b \\ b\sqrt[3]{x^2} \end{cases}$$

$$b\sqrt[3]{x^2} \longrightarrow (a+1)\sqrt[3]{x^2} \xrightarrow[a\sqrt[3]{x^2}+\sqrt[3]{x^2}]{} f'(x)=\frac{2xa}{3\sqrt[3]{x^4}}+\frac{2x}{3\sqrt[3]{x^4}} \to$$

$$\longrightarrow \frac{2\cancel{x}a}{3\cancel{x}\sqrt[3]{x}}+\frac{2\cancel{x}}{3\cancel{x}\sqrt[3]{x}}=\frac{2a+2}{3\sqrt[3]{x}}=1 \to 2a+2=3\sqrt[3]{x} \xrightarrow{x=1}$$

$$\to 2a+2=3 \to 2a=1 \to a=\frac{1}{2} \qquad \to a+b=\frac{5}{2}$$

$$b=\frac{1}{2}+1=\frac{3}{2}$$

سوال ۲

$$f(x)=\frac{|2x-4|}{\sqrt[3]{x^2}} \qquad g(x)=\frac{\sqrt{x^2-4x+4}+\sqrt{3x}}{\sqrt[3]{x^2}}$$

$$f'(1)-2g'(1)=?$$

$$f(x)=\frac{-2x+4}{\sqrt[3]{x^2}} \longrightarrow f'(x)=\frac{-2(\sqrt[3]{x^2})-\frac{2x}{3\sqrt[3]{x^4}}(-2x+4)}{(\sqrt[3]{x^2})^2}$$

$$f'(1)=-2-\frac{2}{3}(2)=-2-\frac{4}{3}=\frac{-10}{3}$$

$$g(x)=\frac{|x-2|+\sqrt{3x}}{\sqrt[3]{x^2}}=\frac{\left(-1+\frac{3}{2\sqrt{3x}}\right)(\sqrt[3]{x^2})-\frac{2x}{3x\sqrt[3]{x}}(2-x+\sqrt{3x})}{(\sqrt[3]{x^2})^2}$$

$$\left(-1+\frac{3}{2\sqrt{3}}\right)=\frac{-2\sqrt{3}+3}{2\sqrt{3}}-\frac{2}{3}(1+\sqrt{3}) \longrightarrow \frac{-6\sqrt{3}+9-4\sqrt{3}-12}{6\sqrt{3}} \to$$

$$\frac{2}{3}+\frac{2\sqrt{3}}{3}=\frac{2+2\sqrt{3}}{3}$$

$$\to \frac{-10\sqrt{3}-3}{6\sqrt{3}}$$

ادامه سوال ۲_

$$f'(1)-2g'(1)=-\frac{10}{3}-2\times\frac{-10\sqrt{3}-3}{3\cdot 4\sqrt{3}}=\frac{-10\sqrt{3}+10\sqrt{3}+3}{3\sqrt{3}}=\frac{3}{3\sqrt{3}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

سوال ۳_

$$f(x)\begin{cases} bx+c & x<a \\ \frac{1}{x} & x\geq 0 \end{cases}$$

مشتق‌پذیر در $\mathbb{R}$ ، $ac=?$

$$\xrightarrow{x=a} ab+c=\frac{1}{a}\rightarrow b=-\frac{1}{a^2}\rightarrow -\frac{1}{a}+c=\frac{1}{a}\rightarrow$$

$$\rightarrow c=\frac{2}{a}\rightarrow \boxed{ac=2}$$

سوال ۴_

$$f(x)=\begin{cases} b & ; x<a \\ b+(x-a)^m & ; x\geq 0 \end{cases}$$

$f'_-(a)=0$

$f'_+(a)\neq 0$

$\xrightarrow{m=0} f'_+(a)=0\rightarrow$ غ ق ق

$\xrightarrow{m=0} f'_+(a)=1\rightarrow$ ق ق $\rightarrow$ فقط یک مقدار $m=1$ امکان دارد

سوال ۵_

$$f(x)=\frac{1}{\sqrt[3]{x-|x|}}\qquad g(x)=\frac{1}{x^3-|x^3|}$$

$$g'(-\sqrt[3]{2})f'g(\sqrt[3]{2})=(fog(-\sqrt[3]{2}))'\Rightarrow fog(x)=\frac{1}{\sqrt[3]{\frac{1}{2x^3}-\left|\frac{1}{2x^3}\right|}}=\frac{1}{\sqrt[3]{\frac{1}{x^3}}}\rightarrow$$

$$\rightarrow\frac{1}{\frac{1}{x}}=x\longrightarrow$$ مشتق x = 1

سوال ۶ -

$f'(۳)-f(۳)=۲ \qquad y=-ax-b$

$$\frac{f(۳)}{f'(۳)} \Rightarrow ?$$

$-a+۳a+b=۲a+b=۲ \longrightarrow f'(۳)=a \qquad \longrightarrow ۲a=-۳$

$\longrightarrow f(۳)=-۳a-b$

$$\longrightarrow a=-\frac{۳}{۲} \longrightarrow \frac{f(۳)}{f'(۳)}=\frac{\frac{۹}{۲}-b}{\frac{۳}{۲}}=-\frac{۱}{۳}$$

سوال ۷

$$g(x)=\frac{۱}{\sqrt[۳]{x^۲-۱}} \qquad f(x)=\left(x\left[x^۲+\frac{۱}{۲}\right]\right)^۳+۱$$

$$(f\circ g)'\left(\frac{۳}{\sqrt{۸}}\right)\longrightarrow g'\left(\frac{۳}{\sqrt{۸}}\right)f'\left(g\left(\frac{۳}{\underbrace{\sqrt{۸}}_{۲}}\right)\right)\longrightarrow g'\left(\frac{۳}{\sqrt{۸}}\right)\times f'(۲)$$

$$\left[x^۲+\frac{۱}{۲}\right]=۴ \longrightarrow f'(x)=\underset{۲}{۳ \cdot ۲x}\longrightarrow ۲۴$$

$$g'(x)=-\frac{۲x}{۳\sqrt[۳]{(x^۲-۱)^۲}}=\frac{-\frac{۳}{\sqrt{۲}}}{\frac{۳}{۴}}=\frac{-۴}{\sqrt{۲}}=\frac{-۴\sqrt{۲}}{۲}$$

$$\frac{-۲\sqrt{۲}\times ۲۴}{-۱۲۸\sqrt{۲}}=۱$$

سوال ۸

$g'(x)=f'(x+۱)+۳f'(۳x+۱۰) \qquad f'(-۱)=f'(۴)=\frac{۳}{۲}$

$g'(-۲)=f'(-۱)+۳f'(۴)$

$$g'(-۲)=\frac{۳}{۲}+\frac{۹}{۲}=۶$$

9-

$$\lim_{h\to 0} \frac{\overbrace{f^2(5-h)}^{t} - 3f(5-h) + 2}{h(5-h)}$$

$f(x) = (x-4)\sqrt[3]{x+3}$

$$\frac{t^2 - 3t + 2}{h(5-h)} = \frac{(t-1)(t-2)}{h(5-h)}$$

$$\frac{f(5-h) - 1}{5-h} \times \frac{f(5-h) - 2}{h} = \frac{f'(5)}{5} = -\frac{5}{12}$$

10-

$6y = 3x + 1 \rightarrow y = \frac{1}{2}x + \frac{1}{6} \longrightarrow m = \frac{1}{2}$

قرینه معکوس $\leftarrow -2$

$2x - 4 = 2$

$x = 1 \rightarrow \boxed{(1, 2)}$