پاسخ نامه

1-

$$g'(\pi/4) = f'\Big(\underbrace{\tan^2\tfrac{\pi}{4} + \sqrt{2}\cos\tfrac{\pi}{4}}_{2}\Big) \times \Big(\underbrace{2\tan\tfrac{\pi}{4}}_{2} \times \underbrace{\big(1+\tan^2\tfrac{\pi}{4}\big)}_{2} + \underbrace{-\sqrt{2}\sin\tfrac{\pi}{4}}_{-1}\Big)$$

$$\Rightarrow \sqrt{3} = f'(2)\times 3 \Rightarrow f'(2) = \sqrt{3}/3$$

2-

$$g'(x) = \frac{-2\sin x}{(2-\cos x)^2} \qquad f'(x) = \frac{(-3\sin x\cos^2 x)(4-\cos^2 x) - (1+\cos^3 x)(\sin(2x))}{(4-\cos^2 x)^2}$$

$$(f-2g)'(x) \Rightarrow \frac{1+\cos^3 x}{4-\cos^2 x} - \frac{4}{2-\cos x} \Rightarrow \frac{1+\cos^3 x - 4(2+\cos x)}{4-\cos^2 x} = \frac{\cos^3 x - 4\cos x}{4-\cos^2 x}$$

$$= \frac{\cos x(\cos^2 x - 4)}{4-\cos^2 x} \Rightarrow (f-2g)'(x) = \sin x \Rightarrow \sin(7\pi/6) = -1/2$$

3-

$$\lim_{x\to 3/4^+} \frac{(4x-3)\sqrt{ax}}{x-3/4} = 2\sqrt{3a} \qquad \lim_{x\to 3/4^-} \frac{(-4x+3)(\sqrt{ax})}{x-3/4} = -2\sqrt{3a}$$

$$\Rightarrow 4\sqrt{3a} = 2\sqrt{12a} = 2\sqrt{6} \Rightarrow a = 1/2$$

4-

$$f(4) + f'(4) = -1 \Rightarrow 2 - 4a + (-a) = -1 \Rightarrow 2 - 5a = -3 \Rightarrow -5a = -3 \Rightarrow a = 3/5$$

$$f'(4) = -a = -3/5$$

5-

$$y = 1/7\, x + 5/7$$

$$y' = \frac{a(3x+1) - 3(ax-1)}{(3x+1)^2} = \frac{a+3}{(3x+1)^2} \Rightarrow \frac{a+3}{(3x+1)^2} = 1/7 \Rightarrow a+3 = (3x+1)^2/7$$

$$\Rightarrow a = \frac{(3x+1)^2 - 21}{7} = \frac{9x^2+1+6x-21}{7}$$

$$x + 5/7 = \frac{ax-1}{3x+1} \Rightarrow 3x^2 + (16-7a)x + 12 = 0$$

$$\Rightarrow 3x^2 + (16 - 9x^2 - 1 - 6x + 21)x + 12 = 0 \Rightarrow -9x^3 - 3x^2 + 36x + 12 = 0 \Rightarrow +3x^3 + x^2 - 12x - 4 = 0$$

$$x^2(3x+1) - 4(3x+1) = 0 \Rightarrow (x^2-4)(3x+1) = 0 \Rightarrow x = 2 \quad a = 49 - 21/7 = 4$$

$$x = \pm 2 \quad x = -1/3$$

$f(۲)=۸+۲a-b=۰ \Rightarrow ۲a-b=-۸ \Rightarrow -۲۴-b=-۸$ -۶

$f'(۲)=۳(۴)+a=۰ \Rightarrow a=-۱۲ \qquad b=-۱۶ \Rightarrow -۱۲+۱۶=-۴$

$۳x^۲+a$

$\lim_{x\to ۰} \frac{f(x)\,f'(x)}{(1-\cos x)^m}=۳۲\sqrt{۲} \Rightarrow \frac{\sin^n(x^۲)\times\left(n\sin^{(n-1)}(x^۲)\right)\times\cos(x^۲)\times ۲x}{(1-\cos x)^m}=۳۲\sqrt{۲}$ -۷

$f^{-1}(x) \Rightarrow \frac{\sqrt{y}+1}{\sqrt{y}-1}=x \qquad x\sqrt{y}-x=\sqrt{y}+1$ -۸

$\Rightarrow \sqrt{y}(x-1)=1+x \Rightarrow \sqrt{y}=\frac{1+x}{x-1} \Rightarrow y=\left(\frac{1+x}{x-1}\right)^۲$

$\Rightarrow y'=۲\left(\frac{1+x}{x-1}\right)\times\left(\frac{-1-1}{(x-1)^۲}\right) \Rightarrow ۲\left(\frac{۳}{1}\right)\times\left(\frac{-۲}{1}\right)=-۱۲$

$f(۰)=1 \Rightarrow \frac{-۸a+۸-1}{-1}=-11 \Rightarrow 11=-۸a+۷ \Rightarrow a=-\frac{1}{۲}$ -۹

$f(-1)=۸\times(-a+1)=-۸a+۸ \qquad ۴=-۸a$

$x=-۲a=-\frac{1}{۲}\times-۲=1 \Rightarrow f'(x)=۳(x^۲+1)^۲\times ۲x\times a(a^{x+1})+a(x^۲+1)^۳$

$\Rightarrow ۳(۴)\times۲\times\frac{\left(-\frac{1}{۲}+1\right)}{\frac{1}{۲}}+-\frac{\frac{1}{۲}(۸)}{-۴}=۸$

$y=\sin^۴x-\cos^۴x=\sin^۲x-\cos^۲x$ -۱۰

$\Rightarrow \frac{\sin\frac{\pi}{۴}\cos\frac{\pi}{۴}-\sin\frac{\pi}{۲}\cos\pi}{\frac{\pi}{۴}-\frac{\pi}{۲}}=\frac{-1\times1\times-1}{\frac{\pi}{۴}-\frac{۲\pi}{۴}}=-\frac{۴}{\pi}$

$\frac{\left(\sin^۲\frac{\pi}{۴}-\cos^۲\frac{\pi}{۴}\right)-\left(\sin^۲\frac{\pi}{۲}-\cos^۲\frac{\pi}{۲}\right)}{-\frac{\pi}{۴}}=\frac{-1(1)}{-\frac{\pi}{۴}}=\frac{۴}{\pi} \Rightarrow \frac{-\frac{۴}{\pi}}{\frac{۴}{\pi}}=-1$