نام و نام خانوادگی ....[illegible]..................   پاسخنامه تشریحی تکلیف شماره ..۲۳.... کلاس ..دوازدهم تجربی..A......

۱	$g(x) = f(\tan^2 x + \sqrt{2}\cos x) \Rightarrow g'(x) = f'(\tan^2 x + \sqrt{2}\cos x)(1+\tan^2 x - \sqrt{2}\sin x)$ $x = \frac{\pi}{4} \Rightarrow g'(\frac{\pi}{4}) = f'(1+\sqrt{2}\times\frac{\sqrt{2}}{2})(1+1-\frac{\sqrt{2}\times\sqrt{2}}{2})$ $\sqrt{3} = f'(2)\times 1 \Rightarrow \boxed{f'(2) = \sqrt{3}}$
۲	$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x}$ $\quad g(x) = \frac{2}{2-\cos x}$ $= \frac{(\cos^2 x - 2\cos x + 4)\cancel{(\cos x+2)}}{(2-\cos x)\cancel{(2+\cos x)}}$ $\quad f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = (f-2g)'(\frac{7\pi}{6})$ $f-2g = \frac{\cos^2 x - 2\cos x + \cancel{4-4}}{2-\cos x} = -\cos x \Rightarrow (f-2g)'(x) = \sin x \Rightarrow \sin\frac{7\pi}{6} = -\frac{1}{2}$
۳	$x > \frac{3}{4} \Rightarrow f(x) = (4x-3)\sqrt{ax} \Rightarrow f'(x) = 4\sqrt{ax} + \underbrace{(4x-3)}_{0}\frac{a}{2\sqrt{ax}} = 4\sqrt{ax}$ $x < \frac{3}{4} \Rightarrow f(x) = -(4x-3)\sqrt{ax} \Rightarrow f'(x) = -(4\sqrt{ax} + \underbrace{(4x-3)}_{0}\frac{a}{2\sqrt{ax}}) = -4\sqrt{ax}$ (اختلاف) $\cancel{2}(4\sqrt{ax}) = \cancel{2}\sqrt{6} \xrightarrow{x=\frac{3}{4}} 4\sqrt{\frac{3}{4}a} = \sqrt{6} \Rightarrow \cancel{16}\times\frac{3}{\cancel{4}}a = 6 \Rightarrow \boxed{a = \frac{1}{2}}$
۴	مماس ⇐ $f(x) = y \quad , f'(x) = y'$ $\quad y + ax = 2 \Rightarrow y = -ax+2$ $\left.\begin{array}{l} f(4) = -4a+2 \\ f'(4) = -a \end{array}\right\} \Rightarrow f(4)+f'(4) = -5a+2 = -1$ $5a = 3 \Rightarrow a = \frac{3}{5}$ $f'(4) = -a = \boxed{-\frac{3}{5}}$
۵	$7y - x = 5 \Rightarrow y = \frac{x}{7}+\frac{5}{7} \Rightarrow y' = \frac{1}{7}$ $y = \frac{ax-1}{3x+1} \Rightarrow y' = \frac{a+3}{(3x+1)^2}$ $\Big\} \Rightarrow \frac{a+3}{(3x+1)^2} = \frac{1}{7} \Rightarrow a = \frac{(3x+1)^2 - 21}{7}$ $\frac{x}{7}+\frac{5}{7} = \frac{ax-1}{3x+1} \xrightarrow{\text{جایگذاری}} \frac{x+5}{\cancel{7}} = \frac{x(3x+1)^2 - 21x - 7}{\cancel{7}(3x+1)} \Rightarrow 9x^3+6x^2-20x-7 = 3x^2+16x+5$ $\Rightarrow 9x^3+3x^2-36x-12 = 0 \xrightarrow{\div 3} x^2(3x+1) - 4(3x+1) = (x^2-4)(3x+1) = 0$

[illegible] $\Rightarrow x = 2 \Rightarrow a = \frac{49-21}{7} = \boxed{4}$

$f(x) = x^3 + ax - b \qquad f'(x) = 3x^2 + a$

[illegible] $x=2 \Rightarrow f(2) = 0 \Rightarrow 8 + 2a - b = 0 \Rightarrow 2a - b = -8$

$f'(2) = 0 \Rightarrow 12 + a = 0 \Rightarrow a = -12$ $\Big\} \Rightarrow b = -16$ ۶

$b - a = -16 - (-12) = \boxed{-4}$

$\lim_{x\to 0} \dfrac{f(x)f'(x)}{(1-\cos x)^m} = 32\sqrt{2} \xrightarrow{*\text{طبق}} \dfrac{\sin^n(x^2)\, x^{2n-1}}{(1-\cos x)^m} = 32\sqrt{2} \xrightarrow{\text{هم‌ارزی}} \dfrac{x^{4n-1}}{x^{2m} \times 2^{-m}} = x^{4n-2m-1} \times 2^m = 32\sqrt{2}$

$f(x) = \sin^n(x^2)$

$f'(0) = \lim_{x\to 0} \dfrac{f(x) - f(0)}{x - 0} = \dfrac{\sin^n(x^2)}{x} \xrightarrow{\text{هم‌ارزی}} \dfrac{x^{2n}}{x} = x^{2n-1}$ $*$

x توان $= 0$

$4n = 2m + 1 \Rightarrow 4n = 12,\ n = 3$

$2^m = 2^{\frac{11}{2}} \Rightarrow m = \frac{11}{2}$

$\Rightarrow 2m + n = 11 + 3 = \boxed{14}$

$f(x) = \dfrac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow y = \dfrac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow \sqrt{x} = \dfrac{y+1}{y-1} \Rightarrow x = \left(\dfrac{y+1}{y-1}\right)^2 \Rightarrow f^{-1}(x) = \left(\dfrac{x+1}{x-1}\right)^2$

$(f^{-1})'(x) = 2\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{-2}{(x-1)^2}\right) \xrightarrow{x=2} 2 \times \dfrac{3}{1} \times \dfrac{-2}{1} = \boxed{-12}$ ۸

$f(x) = (x^2+1)^3(ax+1) \longrightarrow f(0) = 1$

$\qquad \longrightarrow f(-1) = 8(-a+1)$

$\Big\} \to \dfrac{1 - 8(-a+1)}{1} = -11$

$8(-a+1) = 12 \Rightarrow -a + 1 = \dfrac{3}{2}$

$a = -\dfrac{1}{2}$ ۹

$\Rightarrow f(x) = (x^2+1)^3\left(-\tfrac{1}{2}x + 1\right)$

$f'(x) = 3(x^2+1)^2(2x)\left(-\tfrac{1}{2}x+1\right) + (x^2+1)^3\left(-\tfrac{1}{2}\right)$

$\xrightarrow[x=1]{x=-2a} f'(1) = 3 \times 4 \times 2 \times \dfrac{1}{2} + 8\left(-\dfrac{1}{2}\right) = 12 - 4 = \boxed{8}$

$y = \sin x \cos 2x \longrightarrow x = \dfrac{\pi}{4} \Rightarrow \dfrac{\sqrt{2}}{2} \times 0 = 0$

$\qquad \longrightarrow x = \dfrac{\pi}{2} \Rightarrow 1 \times (-1) = -1$ $\Big\} \Rightarrow \dfrac{-1-0}{\frac{\pi}{2} - \frac{\pi}{4}} = -4\pi$

$y = \sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)\underbrace{\cancel{(\sin^2 x + \cos^2 x)}}_{1} \longrightarrow x = \dfrac{\pi}{4} \Rightarrow \dfrac{1}{2} - \dfrac{1}{2} = 0$

$\qquad \longrightarrow x = \dfrac{\pi}{2} \Rightarrow 1 - 0 = 1$ $\Big\} \Rightarrow \dfrac{1-0}{\frac{\pi}{2} - \frac{\pi}{4}} = 4\pi$ ۱۰

$\dfrac{-4\pi}{4\pi} = \boxed{-1}$