۱

$$g'(x) = \underbrace{(\tan^2 x + \sqrt{2}\cos x)'}_{①}\, f'(\underbrace{\tan^2 x + \sqrt{2}\cos x}_{②})$$

$$① \to 2\tan x(1+\tan^2 x) - \sqrt{2}\sin x \xrightarrow{x=\frac{\pi}{4}} 2(2) - 1 = 3$$

$$② \to \tan^2 x + \sqrt{2}\cos x \xrightarrow{x=\frac{\pi}{4}} 1+1 = 2$$

$$g'\left(\frac{\pi}{4}\right) = \sqrt{3} \qquad 3f'(2) = \sqrt{3} \qquad f'(2) = \frac{\sqrt{3}}{3}$$

۲

$$f'\left(\frac{7\pi}{6}\right) - 2g'\left(\frac{7\pi}{6}\right) \to (f-2g)'\left(\frac{7\pi}{6}\right) \to f-2g = \frac{8+\cos^3 x}{4-\cos^2 x} - \frac{4}{2-\cos x}$$

$$= \frac{\cancel{(2+\cos x)}(4+\cos^2 x - 2\cos x)}{(2-\cos x)\cancel{(2+\cos x)}} - \frac{4}{2-\cos x} \to \frac{\cancel{4}+\cos^2 x - 2\cos x \cancel{-4}}{2-\cos x}$$

$$\to \frac{\cos x(\cos x - 2)}{2-\cos x} = -\cos x \xrightarrow{\text{مشتق}} +\sin x \xrightarrow{x=\frac{7\pi}{6}} \sin\frac{7\pi}{6} = -\frac{1}{2}$$

۳

$$\begin{cases} (4x-3)\sqrt{ax} & x \geq \frac{3}{4} \\ (-4x+3)\sqrt{ax} & x < \frac{3}{4} \end{cases}$$

$$f'_+\left(\frac{3}{4}\right) \to 4\sqrt{ax} + \cancel{\frac{a}{2\sqrt{ax}}(4x-3)}^{\,0} = \frac{4\sqrt{3a}}{2}$$

$$f'_-\left(\frac{3}{4}\right) \to -4\sqrt{ax} + \cancel{\frac{a}{2\sqrt{ax}}(-4x+3)}^{\,0} = \frac{-4\sqrt{3a}}{2}$$

$$2\sqrt{3a} - (-2\sqrt{3a}) \Rightarrow \cancel{4}^{\,2}\sqrt{3a} = \cancel{2}\sqrt{6} \xrightarrow{\text{توان ۲}} 12a = 6 \qquad a = \frac{1}{2}$$

۴

$$f(4) + f'(4) = -1 \to -4a + 2 - a = -1 \qquad -5a = -3 \qquad a = +\frac{3}{5}$$

$$y = -\frac{3}{5}x + 2 \xrightarrow[y'=f'(x)]{x=4} f'(4) = -\frac{3}{5}$$

---

۵ ۲ تابع و برخورد هم مماس و پس $D=0$

$$\frac{ax-1}{3x+1} = \frac{x+5}{7} \qquad 7ax - 7 = 3x^2 + x + 15x + 5$$

$$3x^2 + x(16 - 7a) + 12 = 0 \xrightarrow{D=0} 256 + 49a^2 - 224a - 144 = 0$$

$$49a^2 - 224a + 112 = 0 \to 7a^2 - 32a + 16 = 0 \qquad \frac{32 \pm \sqrt{576}}{14}$$

$$\frac{32 \pm 24}{14} \to \frac{8}{4}, \frac{56}{14} \to 2, 4$$

(۹)

$\longrightarrow f'(۲)=۰ \quad ۳x^۲+a=۰ \xrightarrow{x=۲} ۱۲+a=۰ \quad a=-۱۲$

$f(۲)=۰ \longrightarrow ۸+۲\times-۱۲-b=۰ \quad b=-۱۶ \quad b-a \to -۱۶+۱۲ = -۴$

(۸)

$f\left|\begin{matrix}۲\\۹\end{matrix}\right. \longrightarrow f^{-1}\left|\begin{matrix}۹\\۲\end{matrix}\right. \qquad \dfrac{\sqrt{x}+۱}{\sqrt{x}-۱}=۲ \qquad ۲\sqrt{x}-۲=\sqrt{x}+۱$

$\sqrt{x}=۳ \longrightarrow x=۹$

$\xrightarrow{f'(۹)} \dfrac{\left(\frac{۱}{۲\sqrt{x}}\right)(\sqrt{x}-۱)-\left(\frac{۱}{۲\sqrt{x}}\right)(\sqrt{x}+۱)}{(\sqrt{x}-۱)^۲} \xrightarrow{x=۹} -\frac{۱}{۱۲} \to$ شیب $f(x)$

$-۱۲ = f^{-1}(x)$ شیب

(۹)

$\to \left|\begin{matrix}-۱\\-۸a+۸\end{matrix}\right. \left|\begin{matrix}۵\\۱\end{matrix}\right. \qquad m=\dfrac{۱-(-۸a+۸)}{۰-(-۱)}=\dfrac{۸a-۷}{۱}=-۱۱ \qquad ۸a=-۳ \quad a=-\frac{۳}{۸}$

$x=-۲a \longrightarrow -۲\times-\frac{۳}{۸}=\frac{۳}{۴} \longrightarrow f'\left(\frac{۳}{۴}\right) \to ۳(x^۲+۱)^۲(۲x)(ax+۱)+\left(-\frac{۳}{۸}\right)(x^۲+۱)^۳$

(۱۰)

$\left|\begin{matrix}\frac{\pi}{۴}\\۰\end{matrix}\right. \quad \left|\begin{matrix}\frac{\pi}{۲}\\۱\end{matrix}\right. \qquad \xrightarrow{\frac{\pi}{۴}} \sin\frac{\pi}{۴}\cos\frac{\pi}{۲}=۰ \qquad m=\dfrac{۱}{\frac{۱}{\frac{\pi}{۴}}}=\frac{۴}{\pi}$

$\xrightarrow{\frac{\pi}{۲}} \sin\frac{\pi}{۲}\cos\pi=۱$

$\sin^۴x-\cos^۴x \to (\sin^۲x-\cos^۲x)(\cancel{\sin^۲x+\cos^۲x})^{۱} \to (\sin x-\cos x)(\sin x+\cos x)$

$\frac{\pi}{۴} \to \left(\frac{\sqrt{۲}}{۲}-\frac{\sqrt{۲}}{۲}\right)\left(\frac{\sqrt{۲}}{۲}+\frac{\sqrt{۲}}{۲}\right)=۰ \quad \left|\begin{matrix}\frac{\pi}{۴}\\۰\end{matrix}\right. \rightsquigarrow m=\dfrac{۱}{\frac{۱}{\frac{\pi}{۴}}}=\frac{۴}{\pi} \xrightarrow{\text{نتیجه}} (۱)$

$\left|\begin{matrix}\frac{\pi}{۲}\\۱\end{matrix}\right.$

$\frac{\pi}{۲} \to (۱-۰)(۱+۰)=۱$