مریم توکلی    دوازدهم A صبح    تکلیف ۲۳

سوال ۱)

$g(x) = f(\tan^2 x + \sqrt{2}\cos x) \quad g'(\frac{\pi}{4}) = \sqrt{3} \quad f'(2) = ?$

$\to g'(x) = (1+\tan^2 x - \sqrt{2}\sin x)\, f'(\tan^2 x + \sqrt{2}\cos x)$

$x = \frac{\pi}{4} \to g'(\frac{\pi}{4}) = f'(1+1)(1+1-1) = f'(2)\times 1 = \sqrt{3} \to \boxed{f'(2) = \sqrt{3}}$

سوال ۲)

$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} \quad g(x) = \frac{2}{2-\cos x}$

$f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6})$

$\to f(x) = \frac{\cancel{(2+\cos x)}(4+\cos^2 x - 2\cos x)}{\cancel{(2+\cos x)}(2-\cos x)} = \frac{4+\cos^2 x - 2\cos x}{2-\cos x}$

$2g(x) = \frac{4}{2-\cos x} \to f(x) - 2g(x) = \frac{\cancel{4}+\cos^2 x - 2\cos x - \cancel{4}}{2-\cos x}$

$\to \frac{\cos x(\cos x - 2)}{2-\cos x} = -\cos x \to$ مشتق $= \sin x$

$\to \sin(\frac{7\pi}{6}) = \boxed{-\frac{1}{2}}$

سوال ۳)

$f(x) = |4x-3|\sqrt{ax}$

$x = \frac{3}{4}$

$x \to \frac{3}{4}^+ : (4x-3)\sqrt{ax} \xrightarrow[\text{صفرشونده}]{\text{مشتق عامل}} 4\sqrt{ax} \to 4\sqrt{\frac{3a}{4}}$

$x \to \frac{3}{4}^- : (3-4x)\sqrt{ax} \xrightarrow[\text{صفرشونده}]{\text{مشتق عامل}} -4\sqrt{ax} \to -4\sqrt{\frac{3a}{4}}$

$\to \cancel{8}^{4}\sqrt{\frac{3a}{4}} = \cancel{2}\sqrt{9} \to \sqrt{\frac{3a}{4}} = \frac{\sqrt{9}}{4} \to \frac{3a}{\cancel{4}} = \frac{\cancel{9}^{3}}{\cancel{16}_{\cancel{4}\,2}}$

$\to 2a = 1 \Rightarrow \boxed{a = 0/5}$

سوال ۴)

$y+ax=2 \qquad f(4)+f'(4)=-1 \qquad f'(4)=?$

$\to y=-ax+2 \to$ شیب $=-a \to f'(4)=-a$

$x=4 \to y=-4a+2 \qquad -4a+2-a=-1 \Rightarrow -5a=-3 \Rightarrow a=\frac{3}{5}$

$f'(4)=-a=\boxed{-\frac{3}{5}}$

سوال ۵)

$7y-x=5 \qquad y=\frac{ax-1}{3x+1} \qquad a=?$

$\to 7y=x+5 \to m=\frac{1}{7}$

$y'=\frac{\overbrace{a(3x+1)}^{3ax+a}-\overbrace{(ax-1)(3)}^{3ax-3}}{(3x+1)^2}$

$\to \frac{\cancel{3ax}+a-\cancel{3ax}+3}{(3x+1)^2} \to \frac{a+3}{(3x+1)^2}=\frac{1}{7} \to a=\frac{(3x+1)^2-21}{7}$

$\frac{x}{7}+\frac{5}{7}=\frac{ax-1}{3x+1} \longrightarrow \frac{x+5}{\cancel{7}}=\frac{x(3x+1)^2-21x-7}{\cancel{7}(3x+1)}$

$\to 9x^3+6x^2-20x-7=3x^2+16x+5 \to 9x^3+3x^2-36x-12=0$

$\xrightarrow{\div 3} 3x^3+x^2-12x-4=0 \to x^2(3x+1)-4(3x+1)=(x^2-4)(3x+1)$

$\Rightarrow \underset{\text{اصلی یک}}{x=2} \to a=\frac{49-21}{7}=\boxed{4}$

سوال ۶)

$f(x)=x^3+ax+b \qquad b-a$

$x=2 \to$ شیب صفر $\to f'(x)=3x^2+a \xrightarrow{x=2} 12+a=0 \Rightarrow a=-12$

$f(2)=0 \to \underbrace{8-12(2)}_{-16}-b=0 \Rightarrow b=-16 \to -16-(-12)=\boxed{-4}$

سوال ۷)

$f(x)=\sin^n(x^۲)$ $\quad \lim_{x\to ۰}\frac{f(x)f'(x)}{(۱-\cos x)^m}=۴\sqrt[۴]{۲}\to \square$

$۲m+n=?$

$$f'(۰)=\frac{f(x)-\cancel{f(۰)}^{۰}}{x-۰}=\frac{\sin^n(x^۲)}{x}\xrightarrow{\text{هم‌ارزی}}\frac{x^{۲n}}{x}=x^{۲n-۱}$$

$$\square\to \frac{\sin^n(x^۲)\,x^{۲n-۱}}{(۱-\cos x)^m}=۳\sqrt[۴]{۲}\xrightarrow{\text{هم‌ارزی}}\frac{x^{۴n-۱}}{x^{۲m}\times ۲^{-m}}=x^{۴n-۲m-۱}\times ۲^m$$

توان $x$ $=۰$

$۴n=\underbrace{۲m}_{۱۱}+۱\to \underline{n=۳}$

$=۴\sqrt[۴]{۲}\Rightarrow m=\frac{۱۱}{۲}$

$۲m+n=۱۱+۳=\boxed{۱۴}$

سوال ۸)

$$f(x)=\frac{\sqrt{x}+۱}{\sqrt{x}-۱}\to \sqrt{x}=\frac{y+۱}{y-۱}\Rightarrow x=\left(\frac{y+۱}{y-۱}\right)^۲$$

$$\Rightarrow f^{-۱}(x)=\left(\frac{x+۱}{x-۱}\right)^۲\to (f^{-۱})'(x)=۲\left(\frac{-۲}{(x-۱)^۲}\right)\left(\frac{x+۱}{x-۱}\right)$$

$$\xrightarrow{x=۲} ۲(-۲)(۳)=\boxed{-۱۲}$$

سوال ۹)

$f(x)=(x^۲+۱)^۳(ax+۱)$

$x=۰\to y=۱ \qquad x=-۱\to y=\overbrace{(۸)(۱-a)}^{۸-۸a}$

$$-۱۱=\frac{۸-۸a-۱}{-۱}=۸a-۷=-۱۱\to \underline{a=-\frac{۱}{۲}}$$

$x=-۲a\Rightarrow \underline{x=۱}$

$$f'(x)=۳(x^۲+۱)^۲(۲x)\left(-\frac{x}{۲}+۱\right)+(x^۲+۱)^۳\left(-\frac{۱}{۲}\right)$$

$$\to x=۱\to f'(۱)=\underbrace{\left(۳\times ۴\times ۲\times \frac{۱}{۲}\right)}_{۱۲}+\underbrace{(۸)\left(-\frac{۱}{۲}\right)}_{-۴}=\boxed{۸}$$

سوال ۱۰)

$y = \sin x \cos 2x \qquad y = \sin^4 x - \cos^4 x$

$x = \frac{\pi}{4} \longrightarrow \sin\frac{\pi}{4}\cos\frac{\pi}{2} = 0 \qquad x = \frac{\pi}{2} \longrightarrow \sin\frac{\pi}{2}\cos\pi = -1$

$\sin^4\frac{\pi}{4} - \cos^4\frac{\pi}{4} = 0 \qquad \sin^4\frac{\pi}{2} - \cos^4\frac{\pi}{2} = 1$

$y = \sin x \cos 2x \longrightarrow \frac{-1}{\frac{\pi}{2}-\frac{\pi}{4}} = \frac{-1}{\frac{\pi}{4}} = -\frac{4}{\pi}$

$y = \sin^4 x - \cos^4 x \longrightarrow \frac{1}{\frac{\pi}{2}-\frac{\pi}{4}} = \frac{1}{\frac{\pi}{4}} = \frac{4}{\pi}$

$\frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = \boxed{-1}$