آیلا فیروزنیا

۱. $g(x) = f(\tan^2 x + \sqrt{3}\cos x) \rightarrow g'(x) = f'(\tan^2 x + \sqrt{3}\cos x)(1 + \tan^4 x - \sqrt{3}\sin x)$

$x = \frac{\pi}{4} \rightarrow g'(\frac{\pi}{4}) = f'(1 + \sqrt{3} \times \frac{\sqrt{3}}{2})(1 + 1 - \frac{\sqrt{2} \times \sqrt{3}}{2}) \rightarrow \sqrt{3} = f'(3) \times 1$

$f'(3) = \sqrt{3}$

۲. $f(x) = \frac{8 + \cos^3 x}{4 - \cos^2 x} = \frac{\cancel{(2 + \cos x)}(4 + \cos^2 x - 2\cos x)}{\cancel{(2 + \cos x)}(2 - \cos x)} = \frac{\cos^2 x - 2\cos x + 4}{2 - \cos x}$

$f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = (f - 2g)'(\frac{7\pi}{6})$

$f - 2g = \frac{\cos^2 x - 2\cos x + 4 - 4}{2 - \cos x} = \frac{\cos^2 x - 2\cos x}{2 - \cos x} = \frac{\cos x(\cos x - 2)}{2 - \cos x} = -\cos x$

$(f - 2g)'x = \sin x \rightarrow \sin\frac{7\pi}{6} = \frac{-1}{2}$

۳. $x > \frac{3}{4} \quad f(x) = (4x - 3)\sqrt{ax} \rightarrow f'(x) = 4\sqrt{ax} + \cancel{(4x - 3)\frac{a}{2\sqrt{ax}}} = 4\sqrt{ax}$

$x < \frac{3}{4} \quad f(x) = -(4x - 3)\sqrt{ax} \rightarrow f'(x) = -(4\sqrt{ax} + \cancel{(4x - 3)\frac{a}{2\sqrt{ax}}}) = -4\sqrt{ax}$

$\rightarrow 2(4\sqrt{ax}) = 4\sqrt{6} \rightarrow x = \frac{3}{4} \rightarrow 2\sqrt{3a} = \sqrt{6} \rightarrow \sqrt{12a} = \sqrt{6}$

$\rightarrow 12a = 6 \rightarrow a = \frac{1}{2}$

۴. $y = -ax + 2 \rightarrow f(x) = y \rightarrow f'(x) = y'$

$f(4) = -4a + 2$

$f'(4) = -a$

$\rightarrow f(4) + f'(4) = -4a + 2 - a = -5a + 2 = -1$

$5a = 3 \rightarrow a = \frac{3}{5}$

$f'(4) = -a = \frac{-3}{5}$

آیلا فیروزنیا

۵- $7y = x + 5 \to y = \frac{1}{7}x + \frac{5}{7} \to y' = \frac{1}{7} \to \frac{a+3}{(3x+1)^2} = \frac{1}{7}$

$\frac{ax-1}{3x+1} = \frac{1}{7}x + \frac{5}{7} \to \frac{7ax-7}{3x+1} = x+5 \Rightarrow 3x^2 + 15x + x + 5 = 7ax - 7$

$3x^2 + 16x + 5 = 7ax - 7 \to 3x^2 + (16 - 7a)x + 12 = 0$

$7a + 21 = 9x^2 + 6x + 1 \to 7a = 9x^2 + 6x - 20$

$\to 3x^2 + (16 - 9x^2 - 6x + 20)x + 12 = 0 \to 3x^2 + 16x - 9x^3 - 6x^2 + 20x + 12 = 0$

$9x^3 + 3x^2 - 36x - 12 = 0 \xrightarrow{\div 3} 3x^3 + x^2 - 12x - 4 = 0$

$x^2(3x+1) - 4(3x+1) \to (3x+1)(x^2-4) = 0 \to x = 2$ [illegible]

$7a = 36 + 12 - 20 \to a = 4$

۶- $f'(x) = 0 \to$ مماس بر محور x

$\to 3x^2 + a = 0 \to 12 + a = 0 \to a = -12$

$8 - 24 - b = 0 \to b = -16 \to b - a = -16 - (-12) = -4$

۷- $\lim\limits_{x \to 0} \frac{f(x) f'(x)}{(1 - \cos x)^m} = 32\sqrt{2} \to \frac{\sin^n(x^2)\, x^{2n-1}}{(1 - \cos x)^m} = 32\sqrt{2}$

$\to \frac{x^{4n-1}}{x^{2m} \times 2^{-m}} = x^{4n-2m-1} \times 2^m = 32\sqrt{2}$ | $\to 4n = 2m + 1 \to 4n = 12 \to n = 3$

$2^m = 2^{\frac{11}{2}} \to m = \frac{11}{2}$

$f'(0) = \lim \frac{f(x) - f(0)}{x - 0} = \frac{\sin^n(x^2)}{x} \to \frac{x^{2n}}{x} = x^{2n-1}$

$2m + n = 11 + 3 = 14$

آیلا فیروزخنیا

۸. $y=\frac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow \sqrt{x}=\frac{y+1}{y-1} \xrightarrow{\text{توان ۲}} x=\left(\frac{y+1}{y-1}\right)^2$

$f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2 \rightarrow (f^{-1})'(x)=2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right)$

$x=3 \Rightarrow 2\times\frac{3}{1}\times\frac{-2}{1}=-12$

۹. $f(x)=(x^2+1)^3(ax+1) \rightarrow f(0)=1$

$f(-1)=8(-a+1) \rightarrow \frac{1-8(-a+1)}{1}=-11$

$8(-a+1)=12 \rightarrow -2a+2=3 \rightarrow 2a=-1 \rightarrow a=\frac{-1}{2}$

$f(x)=(x^2+1)^3\left(\frac{-1}{2}x+1\right)$

$f'(x)=3(x^2+1)^2(2x)\left(\frac{-1}{2}x+1\right)+\left(\frac{-1}{2}\right)(x^2+1)^3$

$x=-2a \rightarrow x=1 \rightarrow 3\times 4\times 2\times\frac{1}{2}+\frac{-1}{2}\times 8=12-4=8$

۱۰. $y=\sin x\cos^2 x \rightarrow x=\frac{\pi}{2} \rightarrow 1\times -1=-1$

$x=\frac{\pi}{4} \rightarrow \frac{\sqrt{2}}{2}\times 0=0 \rightarrow \frac{0-(-1)}{\frac{\pi}{4}}=\frac{4}{\pi}$

$y=\sin^4 x-\cos^4 x \rightarrow x=\frac{\pi}{4} \rightarrow \frac{1}{4}-\frac{1}{4}=0$

$x=\frac{\pi}{2} \rightarrow 1-0=1 \rightarrow \frac{0-1}{\frac{\pi}{4}}=\frac{-4}{\pi}$

$\frac{\frac{4}{\pi}}{\frac{-4}{\pi}}=\boxed{-1}$