نام و نام خانوادگی ...ستایش معصوم‌زاده... پاسخنامه تشریحی تکلیف شماره ...۱۲... کلاس ...دوازدهم ریاضی A...

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$$g'(x) = f'(\tan^2 x+\sqrt{2}\cos x)(1+\tan^2 x-\sqrt{2}\sin x)$$

$$x=\frac{\pi}{4} \longrightarrow g'\left(\frac{\pi}{4}\right) = f'\left(1+\sqrt{2}\times\frac{\sqrt{2}}{2}\right)\left(1+1-\frac{\sqrt{2}\times\sqrt{2}}{2}\right)$$

$$\sqrt{3} = f'(2)\times 1 \longrightarrow f'(2)=\sqrt{3}$$

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$$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} \qquad g(x) = \frac{2}{2-\cos x}$$

$$\frac{(\cos^2 x-2\cos x+4)(\cos x+2)}{(2-\cos x)(2+\cos x)}$$

$$f-2g \longrightarrow \frac{\cos^2 x-2\cos x+4}{2-\cos x} = -\cos x \longrightarrow (f-2g)'(x) = \sin x$$

$$f'\left(\frac{7\pi}{6}\right)-2g'\left(\frac{7\pi}{6}\right) = (f-2g)'\left(\frac{7\pi}{6}\right)$$

$$\sin\frac{7\pi}{6} = -\frac{1}{2}$$

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$$x>\frac{3}{4} \rightarrow f(x) = (4x-3)\sqrt{ax} \rightarrow f'(x) = 4\sqrt{ax}+(4x-3)\frac{a}{2\sqrt{ax}} = 4\sqrt{ax}$$

$$x<\frac{3}{4} \rightarrow f(x) = -(4x-3)\sqrt{ax} \rightarrow f'(x) = -\left(4\sqrt{ax}+(4x-3)\frac{a}{2\sqrt{ax}}\right) = -4\sqrt{ax}$$

$$\xrightarrow{\text{اختلاف}} \cancel{2}(4\sqrt{ax}) = \cancel{2}\sqrt{6} \xrightarrow{x=\frac{3}{4}} 4\sqrt{\frac{3}{4}a} = \sqrt{6} \rightarrow 16\times\frac{3}{4}a = 6 \rightarrow a=\frac{1}{2}$$

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$$\xrightarrow{\text{مماس}} f(x)=g \quad , \quad f'(x)=g'$$

$$f(4)+f'(4) = -5a+2 = -1 \longrightarrow a=\frac{3}{5}$$

$$f'(4) = -a = -\frac{3}{5}$$

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$$y=\frac{x}{7}+\frac{5}{7} \rightarrow y'=\frac{1}{7}$$

$$y=\frac{ax-1}{3x+1} \rightarrow y' = \frac{a+3}{(3x+1)^2}$$

$$\frac{a+3}{(3x+1)^2} = \frac{1}{7} \longrightarrow a = \frac{(3x+1)^2-21}{7}$$

$$\frac{x}{7}+\frac{5}{7} = \frac{ax-1}{3x+1} \longrightarrow \frac{x+5}{7} = \frac{x(3x+1)^2-21x-7}{7(3x+1)} \Rightarrow 9x^3+6x^2-20x-7 = 3x^2+16x+5$$

$$\rightarrow 9x^3+3x^2-36x-12=0 \xrightarrow{\div 3} x^2(3x+1)-4(3x+1) = (x^2-4)(3x+1) = 0$$

$$\longleftarrow x=2$$

$$\boxed{a=4}$$

$f(x) = x^3 + ax - b \qquad f'(x) = 3x^2 + a$

ماکزیمم $x=2$ $\Rightarrow f(2)=0 \to 8+2a-b=0 \to 2a-b=-8$

$f'(2)=0 \to 12+a=0 \to a=-12$ $\Big\} \to b=-16$

$b-a = -16-(-12) = \boxed{-4}$ ۶

$f'(0) = \lim \dfrac{f(x)-\cancel{f(0)}}{x-0} = \dfrac{\sin^n(x^2)}{x} \xrightarrow{\text{هم‌ارزی}} \dfrac{x^{2n}}{x} = x^{2n-1}$

$\lim\limits_{x\to 0} \dfrac{f(x)f'(x)}{(1-\cos x)^m} = 2\sqrt{2} \longrightarrow \dfrac{\sin^n(x^2)\,x^{2n-1}}{(1-\cos x)^m} = 2\sqrt{2} \xrightarrow{\text{هم‌ارزی}} \dfrac{x^{4n-1}}{\frac{x^{2m}}{2^m}} = x^{4n-2m-1} \times 2^m = 2\sqrt{2}$ ۷

توان صفر $4n = 2m+1$ $\quad 4n=12 \Rightarrow n=3 \quad 2m+1=\ldots$

$2^m = 2^{\frac{3}{2}} \Rightarrow m = \frac{3}{2}$ $\quad$ ۴

$y = \dfrac{\sqrt{x}+1}{\sqrt{x}-1} \to \sqrt{x} = \dfrac{y+1}{y-1} \to x = \left(\dfrac{y+1}{y-1}\right)^2 \to f^{-1}(x) = \left(\dfrac{x+1}{x-1}\right)^2$

$(f^{-1})'(x) = 2\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{-2}{(x-1)^2}\right) \xrightarrow{x=3} 2\times\dfrac{3}{1}\times\dfrac{-2}{1} = -12$ ۸

$f(x) \to f(0)=1$; $\quad f(-1) = 8(-a+1)$ $\Rightarrow \dfrac{1-8(-a+1)}{1} = -11 \Rightarrow -a+1 = \dfrac{3}{2} \to a = -\dfrac{1}{2}$

$f(x) = (x^2+1)^3\left(-\frac{1}{2}x+1\right)$ ۹

$f'(x) = 3(x^2+1)^2(2x)\left(-\frac{1}{2}x+1\right) + (x^2+1)^3\left(-\frac{1}{2}\right)$

$\xrightarrow[x=1]{x=-2a} f'(1) = 3\times4\times2\times\frac{1}{2} + 8\left(-\frac{1}{2}\right) = 8$

$x = \frac{\pi}{4} \longrightarrow \frac{\sqrt{2}}{2}\times 0 = 0$

$x = \frac{\pi}{2} \longrightarrow \ln(-1) = -1$ $\Bigg\} \Rightarrow \dfrac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}} = -\dfrac{4}{\pi}$ ۱۰

$y = \sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)\,\cancel{(\sin^2 x+\cos^2 x)} \longrightarrow x=\frac{\pi}{4} \to \frac{1}{2}-\frac{1}{2} = 0$; $\quad x = \frac{\pi}{2} \to 1-0 = 1$ $\Bigg\} \Rightarrow \dfrac{1-0}{\frac{\pi}{2}-\frac{\pi}{4}} = \dfrac{4}{\pi}$