اهدا ارشدی

(۱)

$$g'(x) = 2(\tan^2 x+1)\times \tan x + (-\sqrt{2}\sin x)\times f'(\tan^2 x+\sqrt{2}\cos x)$$

$$g'\left(\frac{\pi}{4}\right) = (\underbrace{2\times 2\times 1}_{4}) + (\underbrace{-\sqrt{2}\times \frac{\sqrt{2}}{2}}_{-\frac{1}{2}})\times f'(2)$$

$$\longrightarrow 3\times f'(2) = \sqrt{3} \longrightarrow f'(2) = \frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

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(۲)

$$f'\left(\frac{7\pi}{6}\right) - 2g'\left(\frac{7\pi}{6}\right) = ?$$

$$\longrightarrow (f-2g)'\left(\frac{7\pi}{6}\right)$$ ابتدا تابع را می‌سازیم سپس ساده می‌کنیم و در نهایت مشتق آن را می‌گیریم.

$$(f-2g)(x) \rightarrow \frac{8+\cos^3 x}{4-\cos^2 x} - \frac{4}{2-\cos x} = \frac{8+\cos^3 x - 8 - 4\cos x}{4-\cos^2 x}$$

$$\frac{\cos x(\cos^2 x - 4)}{4-\cos^2 x} \rightsquigarrow (f-2g)(x) = -\cos x$$

$$(f-2g)'(x) = -(-\sin x) = \sin x \xrightarrow[x=\frac{7\pi}{6}]{\text{اگر}} \boxed{-\frac{1}{2}}$$

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(۳)

$$\rightarrow (6x-3)\sqrt{ax} \xrightarrow[]{\text{مشتق}} 6\sqrt{ax} + \frac{a}{2\sqrt{ax}}(\underbrace{6x-3}_{})$$

$$\rightarrow (-6x+5)\sqrt{ax} \xrightarrow[]{\text{مشتق}} -6\sqrt{ax} + \frac{a}{2\sqrt{ax}}(\underbrace{-6x+3}_{0})$$

$$4\sqrt{ax}\ \boxed{-}\ (-4\sqrt{ax}) = 8\sqrt{ax} \rightarrow 4\sqrt{3a} \qquad \boxed{a=\frac{1}{2}}$$

$$2\times 4\sqrt{3a} = 2\times\sqrt{9} \longrightarrow \sqrt{3a} = \frac{\sqrt{9}}{2} \longrightarrow 3a = \frac{9}{4} \rightarrow a = \frac{9}{4\times 3}$$

④

$y+ax=2 \longrightarrow y=-ax+2$

$f(4) \longrightarrow y=-4a+2$

$f'(4) \longrightarrow y=-a$

$f(4)+f'(4)=-1 \longrightarrow -4a+2-a=-1 \longrightarrow 3=5a \rightarrow a=\frac{6}{10}$

$f'(4) \longrightarrow y=-a \longrightarrow \boxed{y=-\frac{6}{10}}$

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⑤ چون ۲ منحنی بر هم مماس اند پس معادله تلاقی آن ها باید دارای ریشه مضاعف باشد یعنی $\Delta=0$

$\frac{ax-1}{3x+1}=\frac{5+x}{7} \longrightarrow 7ax-7=3x^2+16x+5$

$3x^2+(16-7a)x+12=0$

$\Delta=0 \longrightarrow (16-7a)^2-(12\times12)=0$

$(16-7a)^2=144 \longrightarrow \begin{cases} 16-7a=12 \rightarrow a=\frac{4}{7} \\ 16-7a=-12 \rightarrow a=4 \end{cases}$

$a=4 \rightarrow 3x^2-12x+12=0 \longrightarrow 3(x-2)^2=0 \rightarrow x=2$

$a=4$ قابل قبول است

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⑥

$f(2)=0 \longrightarrow 2^3+2a-b=0 \longrightarrow 2a-b=-8$

$f'(x)=3x^2+a \xrightarrow{x=2} 12+a=0 \longrightarrow a=-12$

$-24-b=-8 \longrightarrow \boxed{-16=b}$

$b-a \longrightarrow -16-(-12)=-4$

7

$$\lim_{x\to 0} \frac{f(x)\, f'(x)}{(1-\cos x)^m} \longrightarrow \lim_{x\to 0} \frac{2x \cos x^2 \times \sin^{1-n}(x^2)\, y \sin^2(x^2)}{(1-\cos x)^m}$$

$$\longrightarrow \frac{2x\, n \times x^2 \times x^{2-2n} \times x^{2n}}{\left(\frac{x^2}{2}\right)^m} \to \lim_{x\to 0} \frac{2x^5 \times n \times 2^m}{x^{2m}} = 32\sqrt{2}$$

$$\boxed{m = \frac{5}{2}}$$

$$2 \times n \times 4\sqrt{2} = 32\sqrt{2} \longrightarrow n = 4$$

بنابراین $2m + n \longrightarrow \boxed{5 + 4 = 9}$

جواب = ۹

8

$$f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \to y = \frac{\sqrt{x}+1}{\sqrt{x}-1}$$

معادله را بر حسب x بدست آوریم

$$y\sqrt{x} - y = \sqrt{x}+1 \longrightarrow y\sqrt{x} - \sqrt{x} = y+1 \to \sqrt{x}(y-1) = y+1$$

$$\longrightarrow \sqrt{x} = \frac{y+1}{y-1} \longrightarrow x = \left(\frac{y+1}{y-1}\right)^2$$

جای x و y عوض

$$\longrightarrow y = \left(\frac{x+1}{x-1}\right)^2 \longrightarrow f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2$$

مشتق می‌گیریم

$$\longrightarrow 2\left(\frac{x+1}{x-1}\right)\left(\frac{\cancel{x}-1-\cancel{x}-1}{(x-1)^2}\right) \to 2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right)$$

$$\xrightarrow{x=2} 2\left(\frac{3}{1}\right)\left(\frac{-2}{1}\right) = \boxed{-12}$$

9

آهنگ تغییر متوسط

$$\frac{f(0)-f(-1)}{0-(-1)}=\frac{1-8(1-a)}{1}=8a-7=-11$$

$$8a=-4 \longrightarrow a=-\frac{1}{2}$$

آهنگ تغییر لحظه‌ای $\longrightarrow f'(x)=3(x^2+1)^2(2x)\left(-\frac{1}{2}x+1\right)+\left(-\frac{1}{2}\right)(x^2+1)^3$

$$\longrightarrow f'(1)=12-4=8$$

10

$f(x)=\sin x\cos 2x$

$g(x)=\sin^4 x-\cos^4 x$

$$\frac{\dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}}{\dfrac{g(\frac{\pi}{2})-g(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}}=\frac{\overbrace{\sin\frac{\pi}{2}\cos\pi}^{-1}-\overbrace{\sin\frac{\pi}{4}\cos\frac{\pi}{2}}^{0}}{\underbrace{\left(\sin^4\frac{\pi}{2}-\cos^4\frac{\pi}{2}\right)}_{1}-\underbrace{\left(\sin^4\frac{\pi}{4}-\cos^4\frac{\pi}{4}\right)}_{0}}=-1$$