مبینا رضایی

(۱

$$g'(x) = 2(\tan x)(1+\tan^2 x) - \sqrt{2}\sin x \times f'(\tan^2 x + \sqrt{2}\cos x)$$

$$g'(\tfrac{\pi}{4}) = 3 \times f'(2) = \sqrt{3} \longrightarrow f'(2) = \frac{\sqrt{3}}{3}$$

(۲

$$f(x) = \frac{(2+\cos x)(4-2\cos x+\cos^2 x)}{(2+\cos x)(2-\cos x)} \qquad g(x) = \frac{2}{2-\cos x}$$

$$f'(\tfrac{7\pi}{6}) - 2g'(\tfrac{7\pi}{6}) = (f-2g)'(\tfrac{7\pi}{6})$$

$$f-2g = \frac{\cos^2 x - 2\cos x + 4 - 4}{2-\cos x} = -\cos x \xrightarrow{\text{مشتق}} \sin x$$

$$(f-2g)'\frac{7\pi}{6} = \sin\frac{7\pi}{6} = -\frac{1}{2}$$

(۳

$$f(x)\begin{cases} x > \frac{3}{4} \rightarrow (4x-3)\sqrt{ax} \longrightarrow f'(x) = 4\sqrt{ax} + (4x-3)\frac{a}{2\sqrt{ax}} = 4\sqrt{ax} \\ x < \frac{3}{4} \rightarrow -(4x-3)\sqrt{ax} \longrightarrow f'(x) = -\left(4\sqrt{ax} + (4x-3)\frac{a}{2\sqrt{ax}}\right) = -4\sqrt{ax} \end{cases}$$

$$\xrightarrow{\text{اختلاف}} 2 \times 4\sqrt{ax} = 2\sqrt{6} \xrightarrow{x=\frac{3}{4}} 4\sqrt{\tfrac{3}{4}a} = \sqrt{6} \longrightarrow a = \frac{1}{2}$$

(۴

$$\text{خط} \longrightarrow y = 2 - ax \xrightarrow[\text{برابر}]{\text{مقدار و مشتق}} \begin{array}{l} ① \ 2-4a = f(4) \\ ② \ -a = f'(4) \end{array}$$

$$f(4) + f'(4) = 2 - 4a - a = 2 - 5a = -1 \longrightarrow a = \frac{3}{5}$$

$$f'(4) = -a \rightarrow f'(4) = -\frac{3}{5}$$

۵)

شیب‌ها برابر (۱) $\rightarrow \dfrac{1}{7} = \dfrac{a+3}{(3x+1)^2} \longrightarrow a = \dfrac{(3x+1)^2 - 21}{7}$

(۲) مقدارها برابر $\longrightarrow \dfrac{5+x}{7} = \dfrac{ax-1}{3x+1} \xrightarrow{\text{جایگذاری } a} \dfrac{x+5}{7} = \dfrac{x(3x+1)^2 - 21x - 7}{7(3x+1)}$

$$\longrightarrow 9x^3 + 6x^2 - 20x - 7 = 3x^2 + 16x + 5 \longrightarrow 9x^3 + 3x^2 - 36x - 12 = 0$$

$$\xrightarrow{\div 3} x^2(3x+1) - 4(3x+1) = (x^2-4)(3x+1) = 0$$

ناحیه اول $\longrightarrow x = 2 \quad , \quad a = \dfrac{49-21}{7} = \underline{4}$

۶) در نقطه (۲و۰) مقدار و شیب‌ها برابر

$$f'(x) = 3x^2 + a \xrightarrow{(2,0)} 12 + a = 0 \longrightarrow a = -12$$

$$f(x) = x^3 - 12x - b \xrightarrow{(2,0)} 8 - 24 - b = 0 \rightarrow b = -16$$

$$b - a = \boxed{-4}$$

۷)

$$f'(0) = \lim_{x \to 0} \frac{f(x) - \cancel{f(0)}}{x - \cancel{0}} = \frac{\sin^n(x^2)}{x} \xrightarrow{\text{هم‌ارزی}} \frac{x^{2n}}{x} = x^{2n-1}$$

$$\lim \frac{f(x)\,f'(x)}{(1-\cos x)^m} = \frac{\sin^n(x^2) \times x^{2n-1}}{(1-\cos x)^m} \xrightarrow{\text{هم‌ارزی}} \frac{x^{2n} \times x^{2n-1}}{\left(\frac{x^2}{2}\right)^m}$$

$$= \frac{x^{4n-1}}{x^{2m} \times 2^{-m}} = x^{4n-2m-1} \times 2^m = 32\sqrt{2} \xrightarrow{x=0} 2^m = 32\sqrt{2}$$

$$\longrightarrow m = \frac{11}{2} \quad , \quad 4n - 1 = 2m \longrightarrow 4n = 12 \rightarrow n = 3$$

← باید توان‌های x صورت و مخرج برابر باشند

$$2m + n = 14$$

Arman

(۸

$$y=\frac{\sqrt{x}+1}{\sqrt{x}-1} \xrightarrow{f^{-1}} x=\frac{\sqrt{y}+1}{\sqrt{y}-1} \longrightarrow y=\left(\frac{x+1}{x-1}\right)^{۲}$$

$$\left(f^{-1}(x)\right)'=\left(\left(\frac{x+1}{x-1}\right)^{۲}\right)' = ۲\times\frac{x+1}{x-1}\times\frac{-۲}{(x-1)^{۲}} \xrightarrow{x=۲} ۲\times\frac{۳}{۱}\times\frac{-۲}{۱}=-۱۲$$

(۹

$$\frac{f(0)-f(-1)}{0-(-1)}=1 \Rightarrow ۸\times(-a+1)=۸a-۷=-۱۱ \longrightarrow a=-\frac{۱}{۲}$$

$$f'(1)=۳\,(x^{۲}+1)^{۲}(۲x)\left(-\frac{۱}{۲}x+1\right)+\left(\frac{-۱}{۲}\right)\left(x^{۲}+1\right)^{۳}=۱۲+(-۴)=۸$$

(۱۵

$$y=\sin x\cos ۲x$$

$$\lim \frac{f(\frac{\pi}{۲})-f(\frac{\pi}{۴})}{\frac{\pi}{۲}-\frac{\pi}{۴}}=\frac{-1-0}{\frac{\pi}{۴}}=-\frac{۴}{\pi}$$

$$y=\sin^{۴}x-\cos^{۴}x=\left(\sin^{۲}x-\cos^{۲}x\right)\underbrace{\cancel{\left(\sin^{۲}x+\cos^{۲}x\right)}}_{1}$$

$$\lim \frac{f(\frac{\pi}{۲})-f(\frac{\pi}{۴})}{\frac{\pi}{۲}-\frac{\pi}{۴}}=\frac{1-0}{\frac{\pi}{۴}}=\frac{۴}{\pi}$$

$$\frac{-\frac{۴}{\pi}}{\frac{۴}{\pi}}=-1$$