۱)

$$g'\left(\frac{\pi}{4}\right)=\sqrt{3}=f'(\underbrace{\tan^2 x+\sqrt{2}\cos x}_{2})\times\left(\underbrace{2\tan x(1+\tan^2 x)}_{4}-\underbrace{\sqrt{2}\sin x}_{1}\right)\xrightarrow{x=\frac{\pi}{4}}$$

$$3f'(2)=\sqrt{3}\longrightarrow f'(2)=\frac{\sqrt{3}}{3}$$

۲)

$$f'\left(\frac{7\pi}{6}\right)-2g'\left(\frac{7\pi}{6}\right)=(f-2g)'\left(\frac{7\pi}{6}\right)\rightarrow\frac{8+\cos^3 x}{4-\cos x}-\frac{4}{2-\cos x}=\frac{8+\cos^3 x-4(2+\cos x)}{4-\cos^2 x}=$$

$$\frac{\cos x(\cos^2 x-4)}{4-\cos^2 x}=-\cos x=\frac{\sqrt{3}}{2}$$

۳)

$$\text{مشتق راست}=\frac{(4x-3)a}{2\sqrt{ax}}+4\sqrt{ax}\qquad\text{مشتق چپ}=\frac{-a(4x-3)}{2\sqrt{ax}}-4\sqrt{ax}$$

$$\text{اختلاف مشتق چپ و راست}=\frac{(4x-3)a}{\sqrt{ax}}+8\sqrt{ax}\longrightarrow 8\sqrt{\frac{3}{4}a}=2\sqrt{6}\longrightarrow a=\frac{1}{2}$$

۴)

$$\left.\begin{array}{l}f(4)=2-4a\\ f'(4)=-a\end{array}\right\}\longrightarrow 2-5a=-1\rightarrow a=\frac{3}{5}\qquad f'(4)=-a=-\frac{3}{5}$$

۵)

$$\frac{x+5}{7}=\frac{ax-1}{3x+1}\longrightarrow 7(ax-1)=(x+5)(3x+1)\longrightarrow$$

$$3x^2+(16-7a)x+12=0\xrightarrow{\Delta=0}(16-7a)^2=144\longrightarrow 16-7a=\pm 12\longrightarrow$$

$$\begin{cases}a=\frac{4}{7}\rightarrow\frac{x+5}{7}=\frac{\frac{4}{7}x-1}{3x+1}\longrightarrow x=-2\\ a=4\longrightarrow\frac{x+5}{7}=\frac{4x-1}{3x+1}\longrightarrow x=2\end{cases}$$

نقطه برخورد در ناحیه اول $\Leftarrow x>0 \leftarrow a=4$ قابل قبول $\longrightarrow$

۶)

$$\left.\begin{array}{l}8-4a+b=0\longrightarrow 4a-b=8\\ f'(x)=3x^2+a\xrightarrow{x=2}12+a=0\longrightarrow a=-12\end{array}\right\}\longrightarrow b=-32$$

$$b-a=-32-(-12)=-20$$

۷)

$$\sin x\simeq x\longrightarrow x\to 0\quad,\quad 1-\cos x\simeq\frac{x^2}{2}\longrightarrow x\to 0\longrightarrow$$

$$\frac{f(x)\,f'(x)}{(1-\cos x)^2}=\frac{x^{2n}\times 2n\,x^{2n-1}}{\left(\frac{x^2}{2}\right)^m}=\frac{x^{4n-1}\times 2n}{2^{-m}\times x^{2m}}=\div=32\sqrt{2}\rightarrow 4n-1=2m\rightarrow$$

$$2n\times 2^{m}=32\sqrt{2}\rightarrow n\times 2^{\frac{4n+1}{2}}=2^{\frac{11}{2}}\rightarrow n=2\rightarrow m=\frac{7}{2}\longrightarrow 2m+n=7+2=9$$

$$\frac{\sqrt{x}+1}{\sqrt{x}-1}=y \longrightarrow \frac{\sqrt{y}+1}{\sqrt{y}-1}=x \longrightarrow \sqrt{y}=\frac{x+1}{x-1} \longrightarrow y=\frac{(x+1)^2}{(x-1)^2} \longrightarrow \quad (8$$

$$y'=\frac{-4x^2+4}{(x-1)^4} \xrightarrow{x=3} y'=\frac{-16+4}{1}=-12$$

$$\frac{1-8(1-a)}{1}=-11 \longrightarrow 1-8+8a=-11 \longrightarrow 8a=-4 \longrightarrow a=-\frac{1}{2} \longrightarrow -2a=1 \quad (9$$

$$f'(x)=-\frac{1}{2}(x^2+1)^3+3(x^2+1)^2 \times 2x\left(-\frac{1}{2}x+1\right) \longrightarrow f'(x)=-4+12=8$$

(10

$$\frac{-1}{\frac{\pi}{4}}=\frac{-4}{\pi} \quad , \quad \frac{(1-0)-\left(\frac{1}{2}-\frac{1}{2}\right)}{\frac{\pi}{4}}=\frac{4}{\pi}$$

$$\frac{-\frac{4}{\pi}}{\frac{4}{\pi}}=\boxed{-1}$$