فاطمه سلطان الکتابی

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$$\Delta=0 \Rightarrow y_1=\frac{ax-1}{3x+1},\quad y_2=\frac{5+x}{7} \Rightarrow \frac{ax-1}{3x+1}=\frac{5+x}{7} \Rightarrow 7ax-7=3x^2+16x+5$$

$$\Rightarrow 3x^2+(16-7a)x+12=0$$

$$\Delta=0 \Rightarrow (16-7a)^2-4(3)(12)=0$$

$$\Rightarrow (16-7a)^2=144$$

$$\Rightarrow 16-7a=12 \Rightarrow a=\frac{4}{7}\ \times$$

$$16-7a=-12 \Rightarrow a=4\ \checkmark$$

$$a=4 \Rightarrow 3x^2-12x+12=0$$

$$\Rightarrow 3(x-2)^2=0 \Rightarrow x=2\ \checkmark$$

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$$\frac{\dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}}{\dfrac{g(\frac{\pi}{2})-g(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}}=\frac{\overbrace{\sin\frac{\pi}{2}\times\cos\pi}^{-1}-\overbrace{\sin\frac{\pi}{4}\times\cos\frac{\pi}{2}}^{0}}{(\sin^4\frac{\pi}{2}-\cos^4\frac{\pi}{2})-(\sin^4\frac{\pi}{4}-\cos^4\frac{\pi}{4})}=-1$$

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$$\frac{f(0)-f(-1)}{0-(-1)}=\frac{1-8(1-a)}{1}=8a-7=-11 \Rightarrow 8a=-4 \Rightarrow a=-\frac{1}{2}$$

$$f'(x)=3(x^2+1)^3(-\tfrac{1}{2}x+1)+(-\tfrac{1}{2})(x^2+1)^3 \Rightarrow f'(-1)=12-4=8$$

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$$x>\frac{3}{4} \Rightarrow f(x)=(4x-3)\sqrt{ax} \Rightarrow f'(x)=4\sqrt{ax}$$

$$f'_+(\tfrac{3}{4})=4\sqrt{\tfrac{3}{4}a}=4\frac{\sqrt{3a}}{2}=2\sqrt{3a}$$

$$x<\frac{3}{4} \Rightarrow f(x)=-(4x-3)\sqrt{ax} \Rightarrow f'(x)=-4\sqrt{ax}$$

$$\Rightarrow f'_-(\tfrac{3}{4})=-4\sqrt{\tfrac{3}{4}a}=-4\frac{\sqrt{3a}}{2}=-2\sqrt{3a}$$

$$f'_+(\tfrac{3}{4})-f'_-(\tfrac{3}{4})=2\sqrt{6} \Rightarrow 2\sqrt{3a}+2\sqrt{3a}=2\sqrt{6} \Rightarrow (4\sqrt{3a})^2=(2\sqrt{6})^2 \Rightarrow 48a=24 \Rightarrow a=\frac{1}{2}$$

$y = \frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow \sqrt{x}\,y - y = \sqrt{x}+1 \Rightarrow \sqrt{x}\,y - \sqrt{x} = y+1$ (5)

$\Rightarrow \sqrt{x}(y-1) = y+1 \Rightarrow \sqrt{x} = \frac{y+1}{y-1} \Rightarrow x = \left(\frac{y+1}{y-1}\right)^2 \Rightarrow f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2$

$\longrightarrow f'^{-1}(x) = 2\left(\frac{x+1}{x-1}\right)\left(\frac{1(x-1)-1(x+1)}{(x-1)^2}\right) \Rightarrow x = 4 \times (-3) = -12$

$\cancel{f(x) = 2x\,x\cos^2 \sin x^{-1}\, x^2 \quad \lim \frac{f(x) \times f'(x)}{(1+\cos x)^x}}$

(7)

$$\lim_{x\to 0} \frac{x^{2n} \times 2n\, x^{2n-1}}{\left(\frac{x^2}{2}\right)^m} = \lim_{x\to 0} n\, 2^{m+1} \frac{x^{4n-1}}{x^{2m}} = 2^{\frac{11}{2}}$$

$4n - 1 = 2m \longrightarrow n \times 2^{m+1} = 2^{\frac{11}{2}} \Rightarrow n = 2$

$m = \frac{7}{2}$

$\Rightarrow 2m + n = 7 + 2 = 9$

NEGARNOTE