«دوازدهم دفتر جمع‌بندی A» «مبینا عاشق الحسنی»

۱-

$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$ $\qquad g'(\frac{\pi}{4}) = \sqrt{3}$

$g'(x) = 2\tan x(1+\tan^2 x) - \sqrt{2}\sin x$ $\qquad f'(2) = ?$

$g'(x) = (2\tan x(1+\tan^2 x) - \sqrt{2}\sin x) f'(\tan^2 x + \sqrt{2}\cos x)$

$g'(\frac{\pi}{4}) = (2\tan\frac{\pi}{4}(1+\tan^2\frac{\pi}{4}) - \sqrt{2}\sin\frac{\pi}{4}) f'(\tan^2\frac{\pi}{4} + \sqrt{2}\times\cos\frac{\pi}{4})$

$\sqrt{3} = (4 - \sqrt{2}(\frac{\sqrt{2}}{2})) f'(2) \Rightarrow \sqrt{3} = 3f'(2) \Rightarrow f'(2) = \frac{\sqrt{3}}{3}$

۲-

$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x}$ $\qquad g(x) = \frac{2}{2-\cos x}$

$f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = ?$ $\quad (f-2g)'(\frac{7\pi}{6})$

$f(x) = \frac{(2+\cos x)(\cos^2 x - 2\cos x + 4)}{(2-\cos x)(2+\cos x)} = \frac{\cos^2 x - 2\cos x + 4}{2-\cos x}$

$f - 2g = \frac{\cos^2 x - 2\cos x + 4 - 4}{2-\cos x} = \frac{\cos^2 x - 2\cos x}{2-\cos x}$

$f - 2g = \frac{\cos x(\cos x - 2)}{2-\cos x} = -\cos x \Rightarrow (f-2g)'(x) = \sin x$

$\Rightarrow (f-2g)'(\frac{7\pi}{6}) = \sin\frac{7\pi}{6} = -\frac{1}{2}$

۳-

$f(x) = |4x-3|\sqrt{ax}$ $\qquad \frac{3}{4}: \; - \quad + $

$f'_+(\frac{3}{4}) - f'_-(\frac{3}{4}) = 2\sqrt{6} \Rightarrow a = ?$

$f'_+(\frac{3}{4}) \rightsquigarrow f(x) = (4x-3)\sqrt{ax} \to f'(x) = 4\sqrt{ax} \to 4\sqrt{\frac{3}{4}a}$

$f'_-(\frac{3}{4}) \rightsquigarrow f(x) = -(4x-3)\sqrt{ax} \to f'(x) = -4\sqrt{ax} \to -4\sqrt{\frac{3}{4}a}$

$8\sqrt{\frac{3}{4}a} = 2\sqrt{6} \to \frac{64}{4}\times 3a = 4\times 6 \Rightarrow a = \frac{1}{2}$

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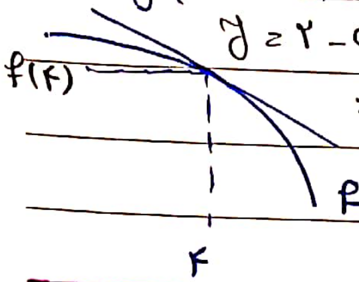
$y+ax=۲$ $\quad f(۴)+f'(۴)=-۱ \Rightarrow f'(۴)=?$

$y=۲-ax$



$f(۴)=۲-۴a$
$f'(۴)=-a$
$\Rightarrow ۲-۴a-a=-۱$

$۲-۵a=-۱$

$۳=۵a \Rightarrow a=\frac{۳}{۵} \Rightarrow f'(۴)=\frac{-۳}{۵}$

-۵

مماس $\begin{cases} ۷y-x=۵ \\ y=\frac{ax-۱}{۳x+۱} \end{cases}$ $\quad a=?$ $\quad \frac{۵+x}{۷}=y$

$\frac{۵+x}{۷}=\frac{ax-۱}{۳x+۱} \Rightarrow ۱۵x+۵+۳x^۲+x=۷ax-۷$

$۳x^۲+(۱۶-۷a)x+۱۲=۰$

$x^۲+\left(\frac{۱۶-۷a}{۳}\right)x+۴=۰$

$\frac{۱۶-۷a}{۳}=۴ \rightsquigarrow (x+۲)^۲=۰$ یا $\quad \frac{۱۶-۷a}{۳}=-۴ \rightsquigarrow (x-۲)^۲=۰$

$۱۶-۷a=۱۲$ $\quad x=-۲$ ✗ $\quad ۱۶-۷a=۱۲$ $\quad x=۲$ ✓ [illegible]

$۴=۷a \to a=\frac{۴}{۷}$ ✗ $\quad ۲۸=۷a \Rightarrow a=۴$

-۶

$f(x)=x^۳+ax-b \to f'(x)=۳x^۲+a$

مماس افقی در $x=۲ \to f'(۲)=۰$ $\quad f'(۲)=۱۲+a=۰ \Rightarrow a=-۱۲$

$b-a=?$ $\quad \to f(۲)=۰ \Rightarrow ۸+۲(-۱۲)-b=۰$

$۸-۲۴-b=۰ \Rightarrow b=۱۶$

$\Rightarrow b-a=۱۶+۱۲=۲۸$

-۷

$f(x)=\sin^n(x^۲)$ $\quad ۲m+n=?$

$\lim_{x\to ۰}\frac{f(x)\,f'(x)}{(۱-\cos x)^m}=۳۲\sqrt{۲}$

$f'(x)=n\sin^{n-۱}(x^۲)\times ۲x\times \cos x^۲$

$\lim_{x\to ۰}\frac{۲x\,n\sin^{n-۱}(x^۲)\times\cos x^۲(\sin^n(x^۲))}{(۱-\cos x)^m}$

$\lim_{x\to ۰}\frac{۲x\times n\times x^۲(۱-\frac{x^۴}{۲})(x^۲)}{(\frac{x^۲}{۲})^m}=۳۲\sqrt{۲}$

۸ـ شیب خط مماس بر $f^{-1}(x)$ در $x=2$

$f(x)=\dfrac{\sqrt{x}+1}{\sqrt{x}-1}$

$f^{-1}\,|\,2 \to ?$ $\longrightarrow$ $f\,|\,?=9 \to 2$

$\dfrac{\sqrt{x}+1}{\sqrt{x}-1}=2 \Rightarrow x=9$

$f'(x)=\dfrac{-2}{(\sqrt{x}-1)^2}\times\dfrac{1}{2\sqrt{x}}$

$f'(9)=\dfrac{-2}{(4)}\times\dfrac{1}{6}=\dfrac{-1}{2}\times\dfrac{1}{6}=\dfrac{-1}{12}$

پس شیب خط مماس بر f^{-1} در $x=2$ برابر -12 (است)

۹ـ

$f(x)=(x^2+1)^3(\frac{-1}{2}x+1)$

$[-1,0] \longrightarrow$ آهنگ متوسط تغییر $=-11 \to \dfrac{f(0)-f(-1)}{1}=-11$

$f'(2x)=?$ $\quad 1-(8(-a+1))=-11$

$f'(-\frac{1}{2}\times 2)=f'(-1)=?$ $\quad 1-(-8a+8)=-11 \Rightarrow 1+8a-8=-11 \Rightarrow a=\frac{-1}{2}$

$f'(x)=3(x^2+1)^2(2x)(\frac{-1}{2}x+1)+(\frac{-1}{2})(x^2+1)^3$

$f'(-1)=3(4)(-2)(\frac{1}{2})+(\frac{-1}{2})8=12-4=8$

۱۰ـ

$[\frac{\pi}{4},\frac{\pi}{2}]$

$f(x)=\sin x\cos 2x$

$g(x)=\sin^4 x-\cos^4 x=\sin^2 x-\cos^2 x=-\cos 2x$

$\dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{g(\frac{\pi}{2})-g(\frac{\pi}{4})}=\dfrac{-1-0}{+1-0}=-1$