نام و نام خانوادگی ...آنیا برزین............... پاسخنامه تشریحی تکلیف شماره ...۲۳... کلاس ...دوازدهم A...

۱	$g(x)= f(\tan^2 x+\sqrt{2}\cos x)$ $g'(x)= (2\tan x+2\tan^3 x-\sqrt{2}\sin x) f'(\tan^2 x+\sqrt{2}\cos x) \qquad g'(\frac{\pi}{4})=\sqrt{3}$ $\Rightarrow g'(\frac{\pi}{4})= (2+2-\sqrt{2}(\frac{\sqrt{2}}{2})) f'(1+\sqrt{2}(\frac{\sqrt{2}}{2})) \Rightarrow 3\times f'(2)=\sqrt{3}$ $\Rightarrow f'(2)=\frac{\sqrt{3}}{3}$
۲	$f(x)=\frac{8+\cos^3 x}{4-\cos^2 x}=\frac{\cancel{(2+\cos x)}(\cos^2 x-2\cos x+4)}{\cancel{(2+\cos x)}(2-\cos x)} \qquad (f-2g)(x)=\frac{\cos^2 x-2\cos x+\cancel{4}-\cancel{4}}{2-\cos x}$ $=\frac{\cos x(\cancel{\cos x-2})}{-(\cancel{\cos x-2})}=-\cos x \qquad (f-2g)'(x)=-\sin x$ $(f-2g)'(\frac{7\pi}{6})=-\sin(\frac{7\pi}{6})=-(-\frac{1}{2})=\frac{1}{2}$
۳	$f'_+(x)=(4\times\sqrt{ax})+(\frac{a}{2\sqrt{ax}}\times(4x-3)) \longrightarrow f'_+(\frac{3}{4})=2\sqrt{3a}$ $f'_-(x)=(-4\times\sqrt{ax})+(\frac{a}{2\sqrt{ax}}\times(3-4x)) \longrightarrow f'_-(\frac{3}{4})=-2\sqrt{3a}$ $f'_+(\frac{3}{4})-f'_-(\frac{3}{4})=\cancel{4}^{2}\sqrt{3a}=\cancel{2}\sqrt{6} \rightsquigarrow 12a=6 \Rightarrow a=\frac{1}{2}$
۴	$y=-ax+2 \longrightarrow f(4)=-4a+2 \qquad f'(4)=-a$ $f(4)+f'(4)=-1 \Rightarrow -4a+2-a=-1 \qquad 3=5a \qquad a=\frac{3}{5}$ $f'(4)=-a=-\frac{3}{5}$
۵	$y=\frac{ax-1}{3x+1} \longrightarrow y'=\frac{a+3}{(3x+1)^2} \qquad y=\frac{x+5}{7}$ $\frac{a+3}{(3x+1)^2}=\frac{1}{7} \longrightarrow 3x^2+16x+12=7ax$ $\xrightarrow{\div x} 3x+16+\frac{12}{x}=7a$ $\frac{ax-1}{3x+1}=\frac{x+5}{7} \longrightarrow 9x^2+6x=7a$ $\longrightarrow 9x^3+3x^2-16x-12=0$ $x=\frac{4}{3} \longrightarrow 7a=9x^2+6x=24 \qquad a=\frac{24}{7}$ $a=4$

$f'(۲)=۰ \rightarrow f'(x)=۳x^۲+a \rightsquigarrow ۰=۱۲+a \quad a=-۱۲$

$f(۲)=۰ \rightarrow f(x)=x^۳-۱۲x-b \rightsquigarrow ۰=۸-۲۴-b \quad b=-۱۶$ ۶

$b-a=-۱۶\overset{+}{\cancel{-}}(-۱۲)=-۴$

$f(x)=\sin^n(x^۲) \longrightarrow f'(x)=۲nx\sin^{n-1}(x^۲)\cos(x^۲)$

$\lim\limits_{x\to ۰}\dfrac{f(x)f'(x)}{(1-\cos x)^m}=\lim\dfrac{۲nx\sin^{۲n-1}(x^۲)\cos(x^۲)}{(1-\cos x)^m}\xrightarrow{\text{هم‌ارزی}}\dfrac{۲nx^{۴n-1}}{\dfrac{x^{۲m}}{۲^m}}=۲^{m+1}n\,x^{۴n-1-۲m}$ ۷

$\rightarrow ۴n-1-۲m=۰ \longrightarrow m=\dfrac{۴n-1}{۲} \qquad ۲^{m+1}n=۳۲\sqrt{۲} \longrightarrow ۲^{\frac{۴n+1}{۲}}=۲^{\frac{۱۱}{۲}}$

$\longrightarrow n=۲ \quad m=\dfrac{۷}{۲} \rightsquigarrow ۲m+n=۷+۲=۹$

ابتدا $f'(۲)$ را پیدا می‌کنیم سپس آن را قرینه معکوس می‌کنیم.

$f(x)=\dfrac{\sqrt{x}+1}{\sqrt{x}-1}$

$f'(x)=\dfrac{\left(\frac{1}{۲\sqrt{x}}\times(\sqrt{x}-1)\right)-\left(\frac{1}{۲\sqrt{x}}\times(\sqrt{x}+1)\right)}{(\sqrt{x}-1)^۲}=\dfrac{-1}{(\sqrt{x})(\sqrt{x}-1)^۲}$ ۸

$f'(۲)=\dfrac{-1}{۳\sqrt{۲}-۴}\xrightarrow{\text{قرینه معکوس}} ۳\sqrt{۲}-۴$

$f(x)=(x^۲+1)^۳(ax+1) \Longrightarrow f'(x)=۳(x^۲+1)(۲x)(ax+1)+(a)(x^۲+1)^۳$

$\dfrac{f(۰)-f(-1)}{۰\overset{+}{\cancel{-}}(-1)}=-۱۱ \rightarrow f(۰)-f(-1)=-۱۱ \rightarrow 1-۸+۸a=-۱۱$

$a=-\dfrac{1}{۲}$ ۹

$f'(-۲a)=f'(1)=۸$

$f(x)=\sin x\cos۲x \Rightarrow f(\frac{\pi}{۲})=1\times-1=-1 \qquad f(\frac{\pi}{۴})=۰$

$g(x)=\sin^۴x-\cos^۴x \Rightarrow g(\frac{\pi}{۲})=1 \qquad g(\frac{\pi}{۴})=۰$

$\dfrac{\dfrac{f(\frac{\pi}{۲})-f(\frac{\pi}{۴})}{\cancel{\frac{\pi}{۲}-\frac{\pi}{۴}}}}{\dfrac{g(\frac{\pi}{۲})-g(\frac{\pi}{۴})}{\cancel{\frac{\pi}{۲}-\frac{\pi}{۴}}}}=\dfrac{f(\frac{\pi}{۲})-\cancel{f(\frac{\pi}{۴})}}{g(\frac{\pi}{۲})-\cancel{g(\frac{\pi}{۴})}}=\dfrac{-1}{1}=-1$ ۱۰