(1) رزین شهریاری

$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$

$g'(x) = (2\tan x(1+\tan^2 x) - \sqrt{2}\sin x) f'(\tan^2 x + \sqrt{2}\cos x)$

$\longrightarrow x = \frac{\pi}{4} \quad \left(2(1+1) - \frac{\sqrt{2}\times\sqrt{2}}{2}\right) f'\left(1 + \frac{\sqrt{2}\times\sqrt{2}}{2}\right) = \sqrt{3}$

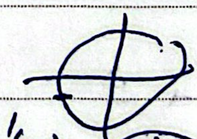
$(4-1) f'(2) = \sqrt{3} \longrightarrow f'(2) = \frac{\sqrt{3}}{3}$

(2)

$$f(x) = \frac{(2+\cancel{\cos x})(\cos^2 x + 2\cos x + \cancel{4})}{(2-\cos x)(\cancel{2+\cos x})} - \frac{\cancel{4}}{2-\cos x}$$

$$f(x) - 2g(x) = \frac{\cos x(\cos x - 2)}{2-\cos x} \Rightarrow f(x) - 2g(x) = -\cos x$$

$f'(x) - 2g'(x) = \sin x$



$\longrightarrow x = \frac{7\pi}{6} \longrightarrow f'(x) - 2g'(x) = -\frac{1}{2}$

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③ $f(x)=|4x-3|\sqrt{ax} \qquad x=\frac{3}{4}$

$+\frac{3}{4}$: $\lim\limits_{x\to\frac{3}{4}^+}\frac{(4x-3)\sqrt{ax}-0}{x-\frac{3}{4}}=4\sqrt{ax}$

$-\frac{3}{4}$: $\lim\limits_{x\to\frac{3}{4}^+}\frac{(3-4x)\sqrt{ax}}{x-\frac{3}{4}}=-4\sqrt{ax}$

$4\sqrt{ax}-(-4\sqrt{ax})=\overset{4}{\cancel{8}}\sqrt{ax}=\overset{1}{\cancel{8}}\sqrt{6}$

$\cancel{4\sqrt{4}} \qquad 4=\sqrt{\frac{6^2\cdot 4}{3\cdot a}} \longrightarrow \boxed{a=\frac{1}{2}}$

④ $y=-ax+2 \qquad A\left|\begin{array}{l}4\\-4a+2\end{array}\right.$

$y'=-a$

$(-a)+(-4a+2)=-1 \qquad -5a+2=-1$

$5a=3 \quad a=\frac{3}{5} \longrightarrow \boxed{f'(4)=-\frac{3}{5}}$

⑤ $y=\frac{1}{7}x+\frac{5}{7}$

$\frac{1}{7}x+\frac{5}{7}=\frac{ax-1}{3x+1}$

مشتق‌ها برابر $\longrightarrow \frac{1}{7}=\frac{a+3}{3x+1} \qquad 3x+1=7a+21$

$7=7a+21$

$-14=7a$

جایگذاری $\boxed{a=-2}$

$\frac{x+5}{\cancel{7}}=\frac{ax-1}{\cancel{7}(a+3)}$

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$(a+3)x+5=ax-1$

$3x+5=-1 \quad \boxed{x=-2}$

۶) $f(x) = x^۳ + ax\cancel{a} - b \xrightarrow[\text{صفر می‌شه هم مشتق}]{\text{۲ مماس شد}}$

$f'(x) = ۳x^۲ + a \xrightarrow[x=۲]{} f'(۲) = ۱۲ + a = ۰ \longrightarrow a = -۱۲$

$f(x) = x^۳ - ۱۲x - b \xrightarrow{x=۲} f(۲) = ۸ - ۲۴ - b \quad b = -۱۴$

$-۱۴ - (-۱۲) = -۴$

۷)

$$\lim \frac{\sin^n(x^۲) \times n \sin^{n-1}(x^۲) \times ۲x \times \cos(x^۲)}{(1-\cos x)^m}$$

$$\xrightarrow{\text{هم‌ارزی}} \lim \frac{x^{۲n} \times n \times (x^۲)^{n-1} \times ۲x \times ۱}{\frac{x^{۲m}}{۲}} = ۴ \times n = ۳۲\sqrt{۲}$$

$n = ۸\sqrt{۲}$

$x^{۲n+۲n-۲+۱} = x^{۲m}$

$۴n - ۱ = ۲m \quad , \quad ۳۲\sqrt{۲} - ۱ = ۲m$

$۳۲\sqrt{۲} - ۱ + ۸\sqrt{۲} = \boxed{۴۰\sqrt{۲} - ۱}$

(1) $f(x)=\dfrac{\sqrt{x}+1}{\sqrt{x}-1}$

$f^{-1}\Big|_{?}^{2} \qquad f\Big|_{2}^{?} \qquad \dfrac{\sqrt{x}+1}{\sqrt{x}-1}=2$

$\cancel{\sqrt{x}+1=2\sqrt{x}-2} \longrightarrow x=9$

$f'(x)=\dfrac{\left(\frac{1}{2\sqrt{x}}\right)(\sqrt{x}-1)-\left(\frac{1}{2\sqrt{x}}\right)(\sqrt{x}+1)}{(\sqrt{x}-1)^2}$

$f'(9)=\dfrac{\left(\frac{1}{6}\right)(2)-\left(\frac{1}{6}\right)(4)}{4}=\dfrac{\frac{1}{3}+\frac{2}{3}}{4}=\dfrac{1}{4}$

$(f^{-1}(9))'=4$

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(9) $x=0 \longrightarrow f(0)=1$

$x=-1 \longrightarrow f(-1)=8(-a+1)=-8a+8$

$\dfrac{-8a+8-1}{-1-0}=-11 \qquad -8a+7=11$

$-8a=4 \qquad a=-\dfrac{1}{2} \longrightarrow \boxed{a=-\dfrac{1}{2}}$

$f'(x)=3(x^2+1)^2(2x)\left(-\frac{1}{2}x+1\right)+\left(-\frac{1}{2}\right)(x^2+1)^3$

$x=1 \rightarrow 3(4)(2)\left(\frac{1}{2}\right)+\left(-\frac{1}{2}\right)(8)$

$f'(1)=12-4=8$

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