رونیا ابوکی

$g'(x)=۲(\tan x)(۱+\tan^۲ x)-\sqrt{۲}\sin x \times f'(\tan^۲ x+\sqrt{۲}\cos x)$ ۱

$g'(\frac{\pi}{۴})=۳\times f'(۲)=\sqrt{۳} \Rightarrow f'(۲)=\frac{\sqrt{۳}}{۳}$

$f(x)=\frac{(۲+\cos x)(۴-۲\cos x+\cos^۲ x)}{(۲+\cos x)(۲-\cos x)} \qquad g(x)=\frac{۲}{۲-\cos x}$ ۲

$f'(\frac{۷\pi}{۶})-۲g'(\frac{۷\pi}{۶})=(f-۲g)'(\frac{۷\pi}{۶})$

$f-۲g=\frac{\cos^۲ x-۲\cos x+۴-۴}{۲-\cos x}=-\cos x \xrightarrow{\text{مشتق}} =\sin x$

$(f-۲g)'(\frac{۷\pi}{۶})=\sin\frac{۷\pi}{۶}=-\frac{۱}{۲}$

$f(x)\begin{cases} x>\frac{۳}{۴} \to (۴x-۳)\sqrt{ax} \to f'(x)=۴\sqrt{ax}+(۴x-۳)\frac{a}{۲\sqrt{ax}} = ۴\sqrt{ax} \quad ۱ \\ x<\frac{۳}{۴} \to -(۴x-۳)\sqrt{ax} \to f'(x)=-(۴\sqrt{ax}+(۴x-۳)\frac{a}{۲\sqrt{ax}}) = -۴\sqrt{ax} \quad ۲ \end{cases}$ ۳

$۱-۲=۲(۴\sqrt{ax})=۲\sqrt{۶} \xrightarrow{x=\frac{۳}{۴}} ۴\sqrt{\frac{۳}{۴}a}=\sqrt{۶} \Rightarrow a=\frac{۱}{۲}$

خط $\to y=۲-ax \qquad f(۴)=۲-۴a \qquad f'(۴)=-a$ ۴

$\Rightarrow f(۴)+f'(۴)=۲-۴a-a=۲-۵a=-۱ \Rightarrow a=\frac{۳}{۵}$

$\Rightarrow f'(۴)=-\frac{۳}{۵}$

دوشا ایوبی

۵

$$\frac{1}{7}=\frac{a+3}{(3x+1)^2} \Rightarrow a=\frac{(3x+1)^3-21}{7}$$

$$\frac{5+x}{7}=\frac{ax-1}{3x+1} \Rightarrow \frac{5+x}{7}=\frac{x(3x+1)^3-21x-7}{7(3x+1)}$$

$$\Rightarrow 9x^3+6x^2-20x-7=3x^2+16x+5 \Rightarrow 9x^3+3x^2-36x-12=0$$

$$\Rightarrow x^3(3x+1)-4(3x+1)=(x^2-4)(3x+1)=0$$

$$\xrightarrow{\text{ناحیه اول}} x=2\ ,\ \boxed{a=4}$$

۶ در نقطه $(2,0)$ عقلا و مشتق‌ها برابر

$$f'(x)=3x^2+a \xrightarrow{(2,0)} 12+a=0 \Rightarrow a=-12$$

$$f(x)=x^3-12x-b \xrightarrow{(2,0)} 8-24-b=0 \Rightarrow b=-16 \qquad \Rightarrow b-a=\boxed{-4}$$

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$$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\frac{\sin^n(x^2)}{x}\xrightarrow{\text{هم‌ارزی}}=\frac{x^{2n}}{x}=x^{2n-1}$$

$$\lim\frac{f(x)f'(x)}{(1-\cos x)^m}=32\sqrt{2}=\frac{\sin^n(x^2)(x^{2n-1})}{(1-\cos x)^m}$$

$$\xrightarrow{\text{هم‌ارزی}}\frac{x^{2n}\times x^{2n-1}}{\left(\frac{x^2}{2}\right)^m}=\frac{x^{4n-1}}{(x^{2m})(2^{-m})}=(x^{4n-2m-1})(2^m)=32\sqrt{2}$$

$$\xrightarrow{x=0} 2^m=32\sqrt{2} \Rightarrow 2^m=2^{\frac{11}{2}} \Rightarrow m=\frac{11}{2}\ ,\ 4n-1=2m=11$$

$$\Rightarrow 4n=12 \Rightarrow n=3 \qquad 2m+n=11+3=\boxed{14}$$

روشنا ایوبی

۸

$$y=\frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow f^{-1} \rightarrow x=\frac{\sqrt{y}+1}{\sqrt{y}-1} \Rightarrow y=\left(\frac{x+1}{x-1}\right)^2$$

$$\left(f^{-1}(x)\right)'=\left(\left(\frac{x+1}{x-1}\right)^2\right)'=2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right)\xrightarrow{x=2} 2\times\frac{3}{1}(-2)$$

$$=\boxed{-12}$$

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$$\frac{f(0)-f(-1)}{0-(-1)}=1-8(-a+1)=8a-7=-11 \Rightarrow a=-\frac{1}{2}$$

$$f'(1)=3(x^2+1)^2(2x)\left(-\frac{x}{2}+1\right)+\left(-\frac{1}{2}\right)(x^2+1)^3=12+(-4)$$

$$=\boxed{8}$$

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$$y=\sin x\cos 2x$$

$$\lim\frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-1-0}{\frac{\pi}{4}}=-\frac{4}{\pi}$$

$$y=\sin^4 x-\cos^4 x=(\sin^2 x-\cos^2 x)\overbrace{(\sin^2 x+\cos^2 x)}^{=1}$$

$$\lim\frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{1-0}{\frac{\pi}{4}}=\frac{4}{\pi}$$

$$\Rightarrow \frac{-\frac{4}{\pi}}{\frac{4}{\pi}}=\boxed{-1}$$