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۱۴۰۳,۱۱,۲۳ تکلیف شماره ۲۳ هانیه نوحیان

دوازدهم دختر A

۴) سوال چهار

خط $y + ax = 2 \xrightarrow[x=4]{}$ مماس ، $f(4) + f'(4) = -1$

$y = -ax + 2$

$\left.\begin{array}{l} f(4) = -4a + 2 \\ f'(4) = -a \end{array}\right\} \Rightarrow -4a + 2 - a = -1$

$-5a = -3$

$a = \frac{3}{5} \Rightarrow \boxed{f'(4) = -\frac{3}{5}}$

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۵) سوال پنج

$y = \frac{x+5}{7}$ منحنی $\Rightarrow \frac{ax-1}{3x-1}$

برای نقطه تماس این دو باید فقط ۱ جواب مشترک داشته باشند $\Rightarrow$

$\frac{x+5}{7} = \frac{ax-1}{3x+1} \Rightarrow (x+5)(3x+1) = 7(ax-1) \Rightarrow$

$3x^2 + (16-7a)x + 12 = 0 \underset{\text{برای مماس بودن}}{\Rightarrow} \Delta = 0 \Rightarrow (16-7a)^2 - 4(3)(12) = 0 \Rightarrow (16-7a)^2 - 144 = 0$

$\Rightarrow (16-7a)^2 = 144 \underset{\text{۲ حالت دارد}}{\Rightarrow}$ ① $16 - 7a = 12 \longrightarrow a = \frac{4}{7}$ چون نقطه در صورت سوال

② $16 - 7a = -12 \longrightarrow a = 4$ در ناحیه اول مختصات

$x_0 > 0$

$x_0 = \frac{7a-16}{6}$

$a = 4 \longrightarrow x_0 = 2$

$\boxed{a = 4}$

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۱) سوال یک

$g(x) = f(\tan^۲ x + \sqrt{۲}\cos x)$

$g'(\frac{\pi}{۴}) = \sqrt{۲}$ $\quad f'(۲) = ?$ $\quad u(x) = \tan^۲ x + \sqrt{۲}\cos x \Rightarrow$

$g(x) = f(u(x)) \Rightarrow g'(x) = f'(u(x)) \cdot u'(x) \Rightarrow u(\frac{\pi}{۴}) \Rightarrow \tan(\frac{\pi}{۴}) = ۱$

$\cos(\frac{\pi}{۴}) = \frac{\sqrt{۲}}{۲}$

$u'(\frac{\pi}{۴}) = (۱)^۲ + \sqrt{۲} \cdot \frac{\sqrt{۲}}{۲} = ۱ + ۱ = ۲ \Rightarrow g'(\frac{\pi}{۴}) = f'(۲) \cdot u'(\frac{\pi}{۴})$

$\Rightarrow u'(x) = \tan^۲ x + \sqrt{۲}\cos x \Rightarrow \frac{d}{dx}(\tan^۲ x) = ۲\tan x \sec^۲ x$

$\frac{d}{dx}(\sqrt{۲}\cos x) = -\sqrt{۲}\sin x \Rightarrow u'(x) = ۲\tan x \sec^۲ x - \sqrt{۲}\sin x$

$\Rightarrow x = \frac{\pi}{۴} \Rightarrow \tan\frac{\pi}{۴} = ۱ \quad \sec\frac{\pi}{۴} = \frac{۱}{\cos\frac{\pi}{۴}} = \sqrt{۲} \Rightarrow \sec^۲\frac{\pi}{۴} = ۲$

$\sin\frac{\pi}{۴} = \frac{\sqrt{۲}}{۲} \quad u'(\frac{\pi}{۴}) = ۲(۱)(۲) - \sqrt{۲} \cdot \frac{\sqrt{۲}}{۲} = ۴ - ۱ = ۳$

$g'(\frac{\pi}{۴}) = f'(۲) \cdot ۳ \Rightarrow \sqrt{۳} = ۳f'(۲) \longrightarrow \boxed{f'(۲) = \frac{\sqrt{۳}}{۳}}$

۲) سوال ۲

$f(x) = \frac{۸ + \cos^۳ x}{۴ - \cos^۲ x} \xrightarrow{\cos x = t} f(t) = \frac{۸ + t^۳}{۴ - t^۲}$

$g(x) = \frac{۲}{۲ - \cos x} \Rightarrow g(t) = \frac{۲}{۲ - t} \quad \frac{d}{dx} f(t) = f'(t) \cdot t'$

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$t = -\sin x \Rightarrow f(x) = -\sin x\, f(t)$

$g(x) = -\sin x\, g(t)$

$f(x) - 2g(x) = -\sin x\,(f(t) - 2g(t)) \xRightarrow{t\ \text{متغیر}} f(t) = \dfrac{8+t^3}{4-t^2} \Rightarrow f'(t) =$

$\dfrac{3t^2(4-t^2) - (8+t^3)(-2t)}{(4-t^2)^2} = 12t^2 - 3t^4 + 16t + 2t^4 = 12t^2 + 16t - t^4$

$f'(t) = \dfrac{12t^2 + 16t - t^4}{(4-t^2)^2}$ $\quad g(t) = \dfrac{2}{2-t} = 2(2-t)^{-1}$

$g'(t) = \dfrac{2}{(2-t)^2}$

$x = \dfrac{7\pi}{6}$

$\cos x = -\dfrac{\sqrt{3}}{2} \Rightarrow t = -\dfrac{\sqrt{3}}{2}$ $\quad \sin x = -\dfrac{1}{2}$

$f - 2g = -\sin x\,(f(t) - 2g(t)) = \dfrac{1}{2}\underbrace{(f(t) - 2g(t))}_{-1} = -\dfrac{1}{2}$

سوال ۳)

$f(x) = |4x-3|\sqrt{ax}$ $\quad x_0 = \dfrac{3}{4}$

$4x - 3 = 0 \longrightarrow x = \dfrac{3}{4}$

سمت راست $\quad x > \dfrac{3}{4}$

$|4x-3| = 4x-3 \Rightarrow f(x) = (4x-3)\sqrt{ax}$

$\Rightarrow 4x - 3 = 0 \longrightarrow f(x_0) = 0 \longrightarrow f'_+(x_0) = (4)\sqrt{a x_0}$

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سمت چپ $x < \frac{3}{4}$

$|4x-3| = -(4x-3) = 3-4x$

$f(x) = (3-4x)\sqrt{ax} \Rightarrow f'_-(x_0) = (-4)\sqrt{ax_0}$

اختلاف شیب ها

$|f'_+ - f'_-| = |4\sqrt{ax_0} - (-4\sqrt{ax_0})| = 8\sqrt{ax_0}$

جایگذاری $x_0 = \frac{3}{4}$

$8\sqrt{ax_0} = 2\sqrt{6}$

$8\sqrt{a\cdot\frac{3}{4}} = 2\sqrt{6} \Rightarrow 4\sqrt{3a} = 2\sqrt{6} \longrightarrow 2\sqrt{3a} = \sqrt{6} \Rightarrow 4(3a) = 6$

$12a = 6$

$a = \frac{1}{2}$

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سوال ۶)

$f(x) = x^3 + ax - b \qquad x = 2 \qquad b - a = ?$

وقتی $x = 2$ ماکس باشد $\Leftarrow f(2) = 0$

$f(2) = 2^3 + 2a - b = 0 \Rightarrow 8 + 2a - b = 0 \Rightarrow b = 8 + 2a$

$f(x) = 3x^2 + a \Rightarrow f(2) = 3(4) + a = 0 \Rightarrow 12 + a = 0 \Rightarrow a = -12$

$b = 8 + 2(-12) = -16 \qquad b - a = -16 - (-12) = \boxed{-4}$

سوال ۱۷

$x \to 0$ تقریب داریم

$\sin t \sim t \qquad 1-\cos x \sim \frac{x^2}{2}$

$f(x) = \sin^n(x^2) \Rightarrow x \to 0 \Rightarrow \sin(x^2) \sim x^2$

$f(x) \sim (x^2)^n = x^{2n}$

$f(x) = n\sin^{n-1}(x^2)\cos(x^2) \cdot 2x$

$\cos(x^2) \sim 1 \qquad \sin(x^2) \sim x^2 \qquad f(x) \sim n(x^2)^{n-1} \cdot 2x = 2n x^{2n-1}$

$f(x) f'(x) \sim x^{2n} \cdot 2n x^{2n-1} = 2n x^{4n-1}$ صورت

$(1-\cos x)^m \sim \left(\frac{x^2}{2}\right)^m = \frac{x^{2m}}{2^m}$

$\frac{f(x) f'(x)}{(1-\cos x)^m} \sim \frac{2n x^{4n-1}}{x^{2m}/2^m} = 2n\, 2^m x^{4n-1-2m}$

$4n-1-2m=0$

$2m = 4n-1 \Rightarrow m = 2n - \frac{1}{2}$

$2n\, 2^m = 32\sqrt{3}$

$2n \cdot 2^{2n-\frac{1}{2}} = 32\sqrt{2}$

$2n \cdot \frac{2^{2n}}{\sqrt{2}} = 32\sqrt{2}$

$2n\, 2^{2n} = 64$

$n\, 2^{2n} = 32$

$n=2 \Rightarrow m = 2n - \frac{1}{2} = 4 - \frac{1}{2} = \frac{7}{2}$

$m+n = \frac{7}{2} + 2 = \frac{11}{2}$

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سوال ۱۸)

$f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1}$ $\quad (f^{-1})'(a) = \frac{1}{f'(x_0)}$

$f(x_0) = a$

$\frac{\sqrt{x}+1}{\sqrt{x}-1} = ۲$ $\quad t = \sqrt{x}$ $\quad \frac{t+1}{t-1} = ۲$ $\quad t+1 = ۲t-۲$

$\boxed{t = ۳}$

$x_0 = t^۲ = ۹$ $\quad t = \sqrt{x}$ $\quad f = \frac{t+1}{t-1}$

$\frac{df}{dt} = \frac{(t-1)-(t+1)}{(t-1)^۲} = \frac{-۲}{(t-1)^۲}$ $\quad \frac{dt}{dx} = \frac{1}{۲\sqrt{x}} = \frac{1}{۲t}$

$f'(x) = \frac{dt}{db} \cdot \frac{dt}{dx} = \frac{-۲}{(t-1)^۲} \cdot \frac{1}{۲t} = \frac{1}{t(t-1)^۲}$

$\Rightarrow f'(۹) = -\frac{1}{۳(۲)^۲} = \boxed{-\frac{1}{۱۲}}$ $\quad (f^{-1})'(۲) = \frac{1}{f'(۹)} = -\frac{1}{۱۲}$

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