$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$ $\quad g'(\frac{\pi}{4}) = \sqrt{3}$ $\quad$ [illegible] $f'(2) = ?$

$(\tan^2 x)' = \frac{2\tan x}{\cos^2 x}$ و $(\sqrt{2}\cos x)' = -\sqrt{2}\sin x$

$g(\frac{\pi}{4}) = f(1+1) \to g(\frac{\pi}{4}) = f(2) \to f'(\frac{\pi}{4}) = \frac{2\tan\frac{\pi}{4}}{\cos^2\frac{\pi}{4}} - \sqrt{2}\sin\frac{\pi}{4} \times$

مشتق داخلی

$\to f'(\frac{\pi}{4}) = 4 - 1 = 3 \to g'(\frac{\pi}{4}) = 3f(2) \xrightarrow{g'(\frac{\pi}{4}) = \sqrt{3}} f(2) = \frac{\sqrt{3}}{3}$

۲-

$$f(x)=\frac{۸+\cos^۳ x}{۴-\cos^۲ x} \qquad g(x)=\frac{۲}{۲-\cos x}$$

$$(f-۲g)'\left(\frac{۷\pi}{۶}\right) \to \frac{۸+\cos^۳ x}{۴-\cos^۲ x}-\frac{۲}{۲-\cos x}=\frac{\cancel{۸}+\cos^۳ x-\cancel{۸}-۴\cos x}{۴-\cos^۲ x} \to$$

$$(f-۲g) \to \frac{\cos x(\cos^۲ x-۴)}{۴-\cos^۲ x}=-\cos x \xrightarrow{(f-۲g)'} \sin x \to$$

$$\to \sin\frac{۷\pi}{۶}=\boxed{-\frac{۱}{۲}}$$

۳-

$$f(x)=|۴x-۳|\sqrt{ax}$$

$$x=\frac{۳}{۴}^{+} \to (۴x-۳)\sqrt{ax} \xrightarrow{f'(x)} ۴\sqrt{ax} \to ۴\sqrt{ax}+۴\sqrt{ax}=۲\sqrt{۶} \to$$

$$x=\frac{۳}{۴}^{-} \to (۳-۴x)\sqrt{ax} \xrightarrow{f'(x)} -۴\sqrt{ax}$$

$$\to ۸\sqrt{ax}=۲\sqrt{۶} \xrightarrow{x=\frac{۳}{۴}} ۸\sqrt{\frac{۳}{۴}a}=۲\sqrt{۶} \to a=\frac{۱}{۲}$$

۴-

$$y+ax=۲ \qquad f(۴)+f'(۴)=-۱$$

$$f(۴) \Rightarrow y=۲-ax \to f(۴)=۲-۴a \; , \; f'(۴)=-a$$

$$۲-۴a-a=-۱ \to -۵a=-۳ \to a=\frac{۳}{۵} \to f'(۴)=-\frac{۳}{۵}$$

ـ۵

$$7y - x = 5 \xrightarrow{\text{مماس بر}} y = \frac{ax-1}{3x+1}$$

$$\downarrow$$

$$y = \frac{x}{7} + \frac{5}{7} \to m = \frac{1}{7}$$

$$y = \frac{ax-1}{3x+1} \xrightarrow{y'} \frac{a(3x+1) - (ax-1)\times 3}{(3x+1)^2} = \frac{a+3}{(3x+1)^2}$$

$$\frac{a+3}{(3x+1)^2} = \frac{1}{7} \to 7(a+3) = (3x+1)^2 \to 7a = 9x^2+6x-20$$

نقطه $\to \frac{ax-1}{3x+1} = \frac{x+5}{7} \to 7(ax-1) = (x+5)(3x+1)$

$$7ax - 7 = 3x^2+16x+5 \to 7a = \frac{3x^2+16x+12}{x} \longrightarrow$$

$$\to 9x^2+6x-20 = \frac{3x^2+16x+12}{x} \to 9x^3+3x^2-36x-12=0$$

$$\xrightarrow{\div 3} 3x^3+x^2-12x-4=0 \to (x^2-4)(3x+1)=0$$

$x = \pm 2 \,,\, -\frac{1}{3} \longrightarrow +2 \leftarrow x$ سوال نقطه ناحیه اول میخواد

$$(3x+1)^2 = 7(a+3) \xrightarrow[x=2]{} 49 = 7a+21 \to a=4$$

$f(x) = x^3 + ax - b$ — ۶

$f(2) = 0 \to 8 + 2a - b = 0 \xrightarrow{a=-12} -16 - b = 0 \to b = -16$

$f'(2) = 0 \to 3x^2 + a = 0 \to 12 + a = 0 \to a = -12$

$b - a = -16 - (-12) = -4$

$f(x) = \sin^n(x^2) \qquad \lim_{x \to 0} \frac{f(x)\, f'(x)}{(1-\cos x)^m} = 32\sqrt{2}$ — ۷

هم‌ارزی

$f(x) \sim x^{2n} \xrightarrow{f'(x)} 2n\, x^{2n-1}$

$(1-\cos x)$ هم‌ارزی $\to \left(\frac{x^2}{2}\right)^m$

$\frac{x^{2n} \times 2n\, x^{2n-1}}{\left(\frac{x^2}{2}\right)^m} = 2n \times 2^m \times x^{-2m+4n-1} \to x = 0 \to -2m + 4n - 1 = 0$

$\to m = \frac{7}{2}$

$$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \longrightarrow f^{-1}(2) \Rightarrow f(x)=2 \longrightarrow$$ ۸

$$\longrightarrow \frac{\sqrt{x}+1}{\sqrt{x}-1}=2 \longrightarrow \sqrt{x}+1=2\sqrt{x}-2 \longrightarrow \sqrt{x}=3 \longrightarrow x=9$$

$$f'(x)=\frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1)-\frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{(\sqrt{x}-1)^2}=\frac{\left(\frac{\sqrt{x}}{2\sqrt{x}}-\frac{1}{2\sqrt{x}}\right)-\left(\frac{\sqrt{x}}{2\sqrt{x}}+\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x}-1)^2}$$

$$\frac{\frac{1}{\sqrt{x}}-\frac{2}{\sqrt{x}}}{(\sqrt{x}-1)^2} \longrightarrow f'(9)=-\frac{1}{12}$$

$$f(x)=(x^2+1)^3(ax+1) \quad [-1,0]$$ ۹

$$\frac{f(0)-f(-1)}{1}=1-8(-a+1)=8a-7=-11$$

$$8a=-4 \rightarrow a=-\frac{1}{2} \longrightarrow x=-2a \rightarrow x=1$$

$$f'(x)=3(x^2+1)^2(2x)(\overset{-\frac{1}{2}}{a}) \longrightarrow 3(8)(2)\left(-\frac{1}{2}\right)=-24$$

۵-

$f(x)$

$y = \sin x \cos 2x \qquad \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$h(x)$ $y = \sin^4 x - \cos^4 x \rightarrow (\sin^2 x - \cos^2 x)\underbrace{\cancel{(\sin^2 x + \cos^2 x)}}_{1}$

$$\frac{f\left(\frac{\pi}{2}\right) - f\left(\frac{\pi}{4}\right)}{h\left(\frac{\pi}{2}\right) - h\left(\frac{\pi}{4}\right)} = \frac{-1}{1} = -1$$