سیده ثنا بنی طبا   تکلیف شماره ۲۳   دوازدهم دفتر - A

① $g(x) = f(\tan^2 x + \sqrt{2}\cos x)$ و $g'(\frac{\pi}{4}) = \sqrt{3}$

$g'(x) = (2\tan x \times \frac{1}{\cos^2 x} - \sqrt{2}\sin x) f'(\tan^2 x + \sqrt{2}\cos x)$

$g'(\frac{\pi}{4}) = (2\times 1\times 2 - 1) f'(1+1) = 3f'(2) = \sqrt{3} \Rightarrow f'(2) = \frac{\sqrt{3}}{3}$

② $f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) \Rightarrow (f-2g)'(\frac{7\pi}{6}) \Rightarrow \sin(\frac{7\pi}{6}) = \frac{-1}{2}$ $\boxed{\frac{-1}{2}}$

$\frac{8+\cos^3 x}{4-\cos^2 x} - \frac{4}{2-\cos x} = \frac{8+\cos^3 x - 8 - 4\cos x}{4-\cos^2 x} = \frac{\cos x(\cos^2 x - 4)}{(2-\cos x)(2+\cos x)} = -\cos x$

$\Rightarrow$ مشتق می‌گیریم $\rightarrow -(-\sin x) = \sin x$

③ $f(x) = |4x-3|\sqrt{ax}$ $\frac{3}{4}$: $-$ | $+$ $(x=\frac{3}{4}) \rightarrow 8\sqrt{ax} = 2\sqrt{6}$

$(\frac{3}{4})^+ \rightarrow (4x-3)\sqrt{ax} \xrightarrow{\text{مشتق می‌گیریم}} 4\sqrt{ax} + \frac{a}{2\sqrt{ax}} \times 0 = 4\sqrt{ax}$ $8\sqrt{\frac{3}{4}a} = 2\sqrt{6}$

$(\frac{3}{4})^- \rightarrow (-4x+3)\sqrt{ax} \longrightarrow -4\sqrt{ax} + \frac{a}{2\sqrt{ax}} \times 0 = -4\sqrt{ax}$ $\Rightarrow a = \frac{1}{2}$

④ $y = -ax + 2$ $f(x) + f'(x) = -1$ $(x=4)$ $f'(4) = -a = -0.6$

(4)

$y = -ax + 2 \qquad f(x) + f'(x) = -1 \quad (x = 4) \qquad f'(4) = -a = \underline{-0.6}$

$-4a + 2 - a = -1 \longrightarrow 2 - 5a = -1 \longrightarrow a = 0.6$

(5)

$7y = x + 5 \rightarrow y = \frac{1}{7}x + \frac{5}{7} \qquad y = \frac{ax-1}{3x+1}$

$\frac{a(3x+1) - 3(ax-1)}{(3x+1)^2} = \frac{1}{7} \Rightarrow \frac{3+a}{(3x+1)^2} = \frac{1}{7} \Rightarrow 9x^2 + 6x + 1 = 7a + 21$

$9x^2 + 6x + 1 - 21 - 7a = 0 \Rightarrow 9x^2 + 6x - 20 - 7a = 0 \qquad \Delta = 0 \Rightarrow 36 - 4 \times 9 \times (-20 - 7a) = 0$

$-20 - 7a = 1 \Rightarrow \underline{a = -3}$

(6) وقتی بر محور xها مماس است، $x = \alpha$ خود تابع را صفر می‌کند و مشتق را نیز صفر می‌کند.

$8 + 2a - b = 0 \qquad f'(x) = 3x^2 + a$

$12 + a = 0 \qquad b - a = -16 + 12 = \underline{-4}$

$\Rightarrow a = -12 , b = -16$

$f(x) = \sin^n(x^2) \qquad \lim_{x \to 0} \frac{f(x) f'(x)}{(1 - \cos x)^m} = 32\sqrt{2} \qquad \sin^n(x^2) \simeq x^{2n} \xrightarrow{\text{مشتق}} 2n\, x^{2n-1}$

$\xrightarrow{\text{هم‌ارزی}} \frac{x^{2n} \times 2n x^{2n-1}}{\left(\frac{x^2}{2}\right)^m} = 2n \times 2^m \times x^{4n-1-2m}$

$\longrightarrow$ توان x صفر $4n - 2m = 1$

$\longrightarrow n \times 2 \times 2^m = 2^{\frac{11}{2}}$

$\begin{cases} n = 2 \\ m = \frac{7}{2} \end{cases}$

$2m + n = \underline{9}$

(8)

$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1}$ $\quad (f^{-1}(2))'=?$ $\quad \frac{1}{f'(9)}=\frac{1}{-\frac{1}{12}}=-12$

$\frac{\sqrt{x}+1}{\sqrt{x}-1}=2 \rightarrow \sqrt{x}+1=2\sqrt{x}-2 \rightarrow \sqrt{x}=3 \rightarrow x=9$

$f'(x)=\frac{\frac{1}{\sqrt{x}}-\frac{2}{\sqrt{x}}}{(\sqrt{x}-1)^2} \Rightarrow f'(9)=\frac{\frac{-1}{6}}{\frac{2}{1}}=-\frac{1}{12}$

(9)

$f(x)=(x^2+1)^3(ax+1)$ $\quad \frac{f(0)-f(-1)}{0-(-1)}=f(0)-f(-1)=1-1(-a+1)=1a-0=-11$

$-2a=11$ $\quad f'(1)=3^3(2x)(x^2+1)^2\underbrace{\left(-\frac{1}{2}+1\right)}_{\frac{1}{2}}+\left(-\frac{1}{2}\right)(x^2+1)^3=12-6=6$ $\quad a=-\frac{1}{2}$

(10)

$f(x)=\sin x\cos 2x$

$g(x)=(\sin^2 x-\cos^2 x)\underbrace{(\sin^2 x+\cos^2 x)}_{1}$

$\Rightarrow \frac{f(\frac{\pi}{2})-f(\frac{\pi}{6})}{g(\frac{\pi}{2})-g(\frac{\pi}{6})}=\frac{-1-0}{1-0}=\frac{-1}{0}$